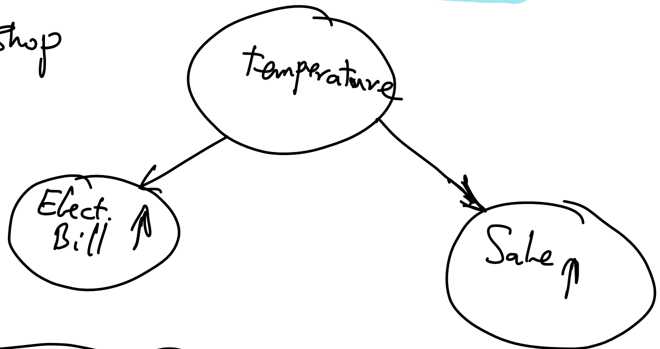
Causality: As a double-edge Sword

Correlation is not Causation.

Observation can be different from action.

e.g., Ice cream Shop



How to Model Causality? Various Models

— Structural Causal Model

→ A program with a sequence for generating a distribution from the indep. random noise variables $\{U_1 \dots U_K\}$

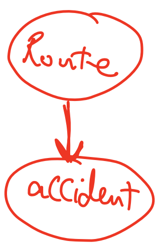
noise Variables

$U \leftarrow B(1/2)$; $U_1 \leftarrow B(1/3)$; $U_2 \leftarrow B(1/3)$

If $U = 0$ route ← R_1 else R_2

If $U = 0$, $U_1 = 1$: $Y = 1$ // late forward
else $Y = 0$ // on time

else
if $U_2 = 1$: $Y = 1$
else
$Y = 0$



Formal Definition:

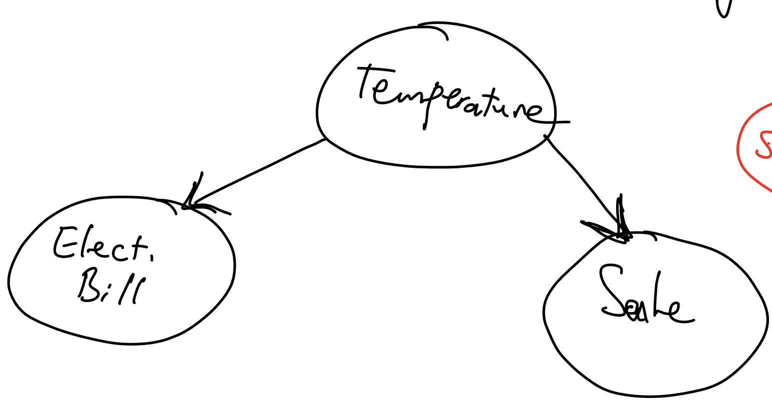
A SCM M is given by a set of random variables $X_1 \dots X_d$ and the functions of value allocations

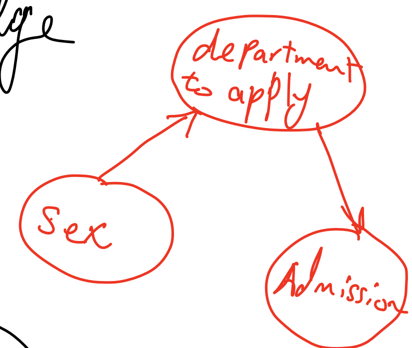
$$X_i = f(\underbrace{P_i}_{\text{Parents of } X_i}, \underbrace{U_i}_{\text{random noises}})$$

* SCM $\underbrace{\text{provides}}_{\text{Sample from}}$ the joint distribution

Structural Causal graph: is a directed graph that represents the SCM:

- $\forall X_i$, add a node
- $\forall (P_i, X_i)$ add an edge

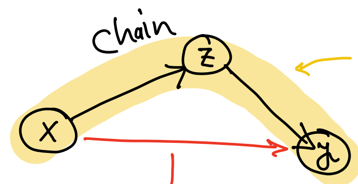




- SCGraph is a DAG
- SCGraph is a Baysian Network.

Chain (Mediator)

Causal
Indirect Effect



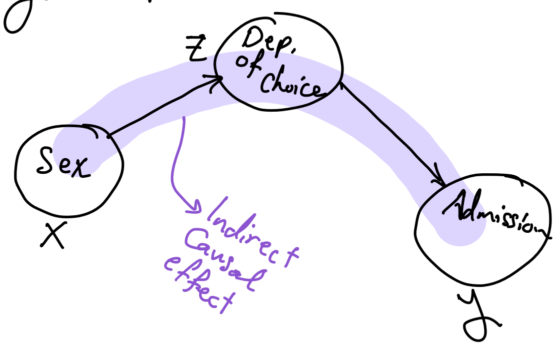
Indirect Causal effect from x to y

Direct effect
Causal

$Cov(X, y) \mid Z \leftarrow$ direct effect

$Cov(X,y) - Cov(X,y) \mid Z \leftarrow$ Indirect effect

Berkeley Example:

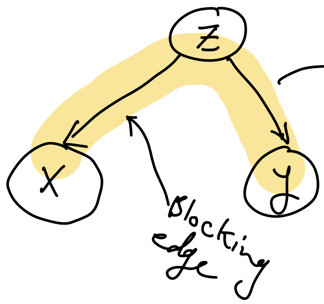


$X \perp\!\!\!\perp y \mid Z$

→ No direct effect

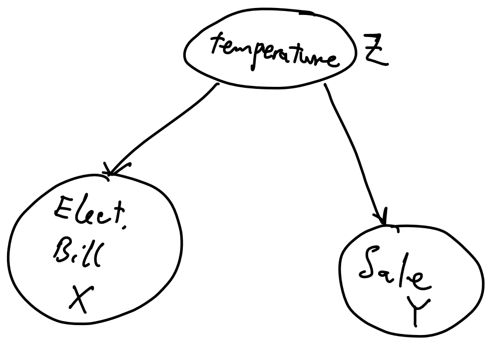
$Cov(X,y) \mid Z = 0$

Fork



The Path Shows z is a Confounder

Ice Cream Shop example

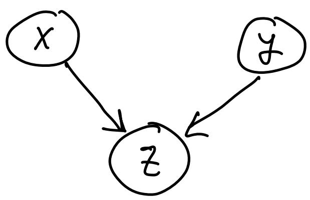


$Cov(X, y) \neq 0$ There is a Correlation between (X, y)

$$\boxed{X \perp\!\!\!\perp Y \mid Z}$$

Conditioning on Z does not show any Causal effect

Collider



$X \perp\!\!\!\perp y$

$X \not\perp\!\!\!\perp y \mid Z$

Measurement Bias (if mistakenly Condition on Z)

Collider Bias $\in$ Sampling Bias

$$\begin{tikzcd}
\text{Responders} \arrow[dr] & & \text{Interest In Surveys} \arrow[dl] \\
& \text{Those who Respond} &
\end{tikzcd}$$

Berkson's Law

Can GRE show if a Student is Strong.

↳ Is there a Correlation b/. GRE Score & Performance.

▷ Study: High correlation b/ the two

Performance → | Grad-School GPA
| After-grad- Career

$$\begin{tikzcd}
\text{GRE} \arrow[dr] & & \text{Performance} \arrow[dl] \\
& \text{Admission} &
\end{tikzcd}$$

Interventions & Causal Effect.

- Do-operator

$U \leftarrow B(1/2); U_1 \leftarrow B(1/3); U_2 \leftarrow B(1/3)$

route $\leftarrow R_1$

If $U = \emptyset$ route $\leftarrow R_1$ else R_2

If $U = \emptyset$, $U_1 = 1$: $Y = 1$ // late for work
else $Y = \emptyset$ // on time

else
if $U_2 = 1$: $Y = 1$
else
$Y = \emptyset$

The do-operator

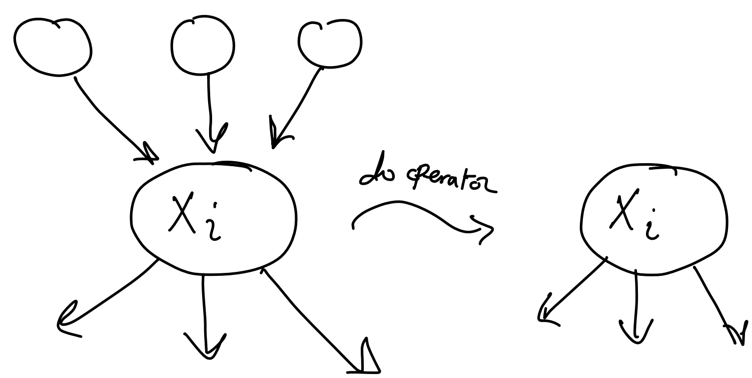
replace

$$X_i = f(P_i, U_i)$$

with

$$X_i = x$$

Visually



$$P(Y \mid do(X)) \underset{\text{not necessarily}}{\neq} P(Y \mid X)$$

Average Treatment Effect:

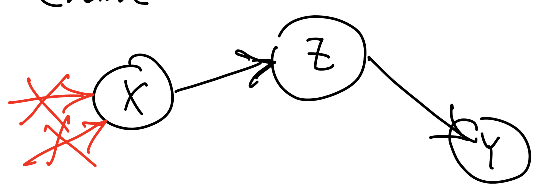
If you change the value of a treatment X, what is the average effect on Y.

$$E[Y \mid do(X=1)] - E[Y \mid do(X=0)]$$

Confounding: Shows the disagreement between the causal and associational statements

→ when $P(Y \mid do(x)) \neq P(Y \mid X)$

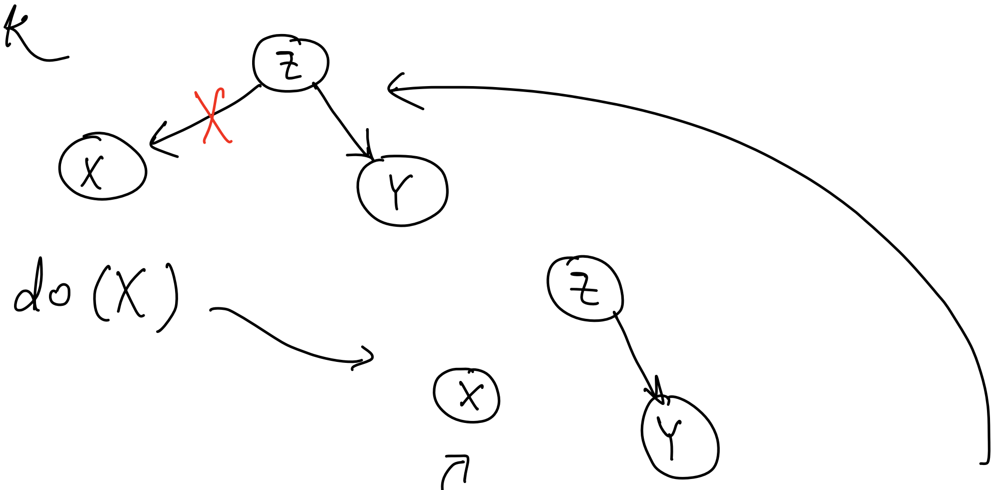
- Chain



$do(X)$

$$P(y \mid do(x)) = P(y \mid X)$$

- Fork



$P(Y|do(X)) \neq P(y|X)$

Confounding Effect: No Causal information shown as Correlation

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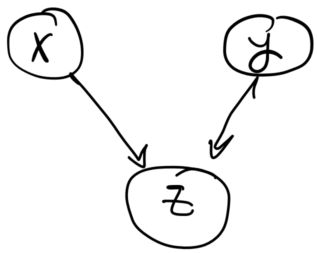


$P(Y|do(x)) = P(y|x)$

- Chain: $X, Y \mid Z$ removes indirect Causal effect
- Fork: $X, Y \mid Z$ removes the Confounding effect
- Collider: $X, Y \mid Z$ Creates Collider bias

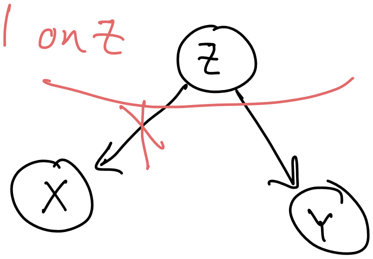
- Backdoor Path: enables the flow of information which is **not Causal**

  ↳ when a Path from X to Y has a backward edge to **X**

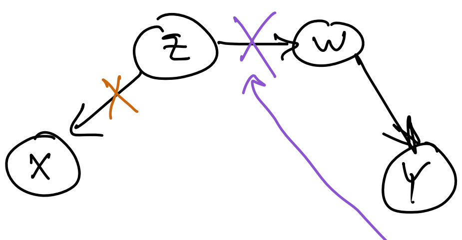
e.g.,

- To remove the Confounding effect, we need to **"block"** the backdoor paths

Condition " "
Controll on Z



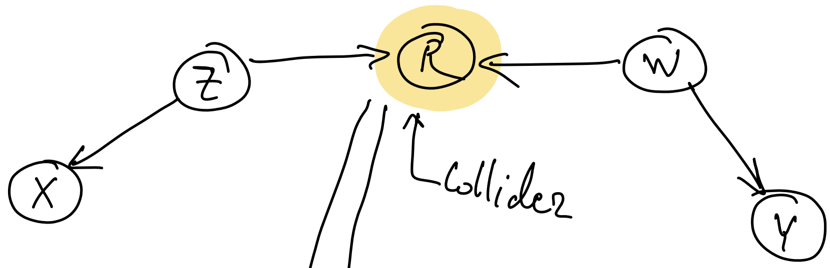
$X, Y \mid Z$



$X, y \mid Z$

OR

$X, Y \mid W$



There is no flow of information from X to Y bcz, R is a collider

Mistakenly conditioning on R, creates the collider bias

When studying causal effect:

Step1: Identify the backdoor paths and block them

$\Rightarrow$ ALL of the remaining correlations are causal. That is,

$$P(Y \mid do(X)) = P(Y \mid X)$$

Step 2:

(A) There is no direct effect from X to Y

→ $X \perp\!\!\!\perp Y \mid C, \mathcal{E}$

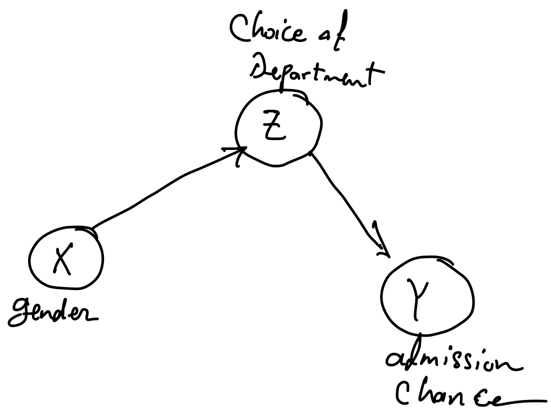
confounding blocking variable

chain Variables

(B) ALL indirect Causal effects are Socially acceptable,

⇒ There is NOT Discriminatory

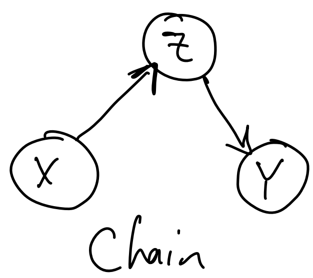
e.g.,

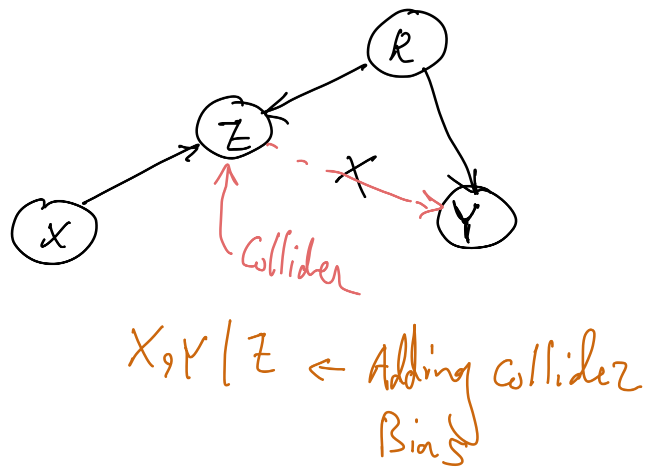


✓ non-discriminatory ✓

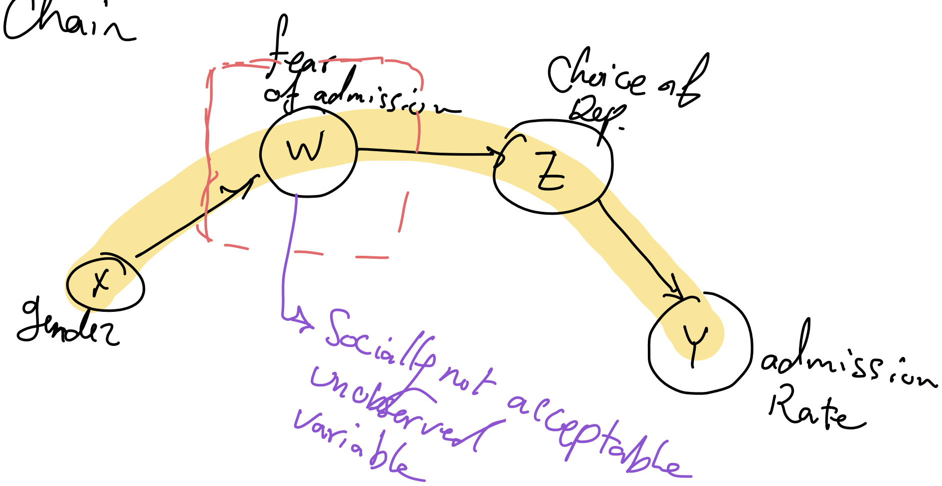
The above argument Can be misleading.

Condition (A) Can be violated, if (Z) is a Collider through an unobserved variable





- Condition (B) Can be violated, if there are unobserved variables in the Chain



- Systematic Policies: Design Policies that change the System in benefit of One group.

discriminatory

→ e.g., give less money to the Dep. that women apply more to.