[Traveling Salesman Problem]

Given a complete weighted graph, Find the complete cycle with min cost

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* General TSP has no approx. Algorithm.

* Metric TSP
  - 2-approx. Alg.
  - 1.5-approx. Alg.
  - best known $4/3$-approx.

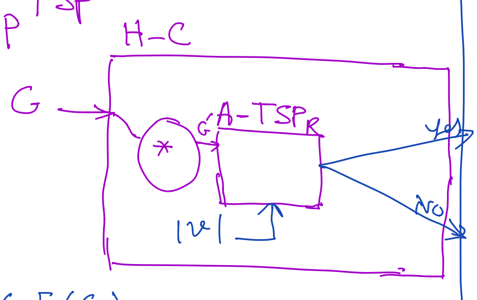
* Euclidian TSP ← $(1+\epsilon)$-Approx. (PTAS)

- General TSP has no approx. Alg. unless P=NP

* Assume A is an approx. Alg. w/t ratio $R = A/O$

We show a reduction from a known NP-Complete problem, X such that the A can solve X exactly

$H\text{-}C \leq_p TSP$

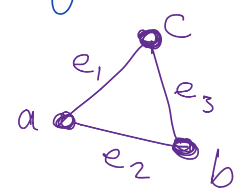


$\forall (u,v) \in E(G)$
add $w(u,v) = 1$ to G'

$\forall (u,v) \notin E(G)$
$w(u,v) = |v| * R$ to G'

Metric TSP:

Edge weights satisfy Triangular inequality



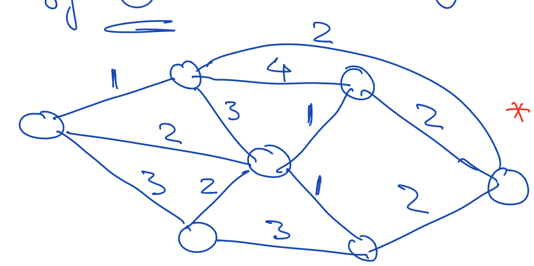
$e_3 \leq e_1 + e_2$

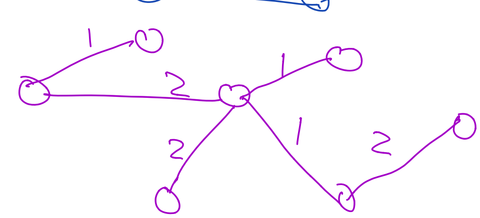
2-approx. Alg. for MTSP

Eularian Circuit:
a cycle that passes through each edge exactly once

In an Eularian Graph all the vertices have an even degree

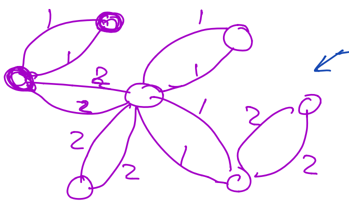
Step 1:
Find the Min. Spanning Tree of G





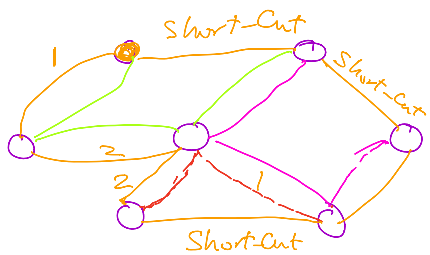
* Assume G is a complete graph :)

Step 2: duplicate the edges

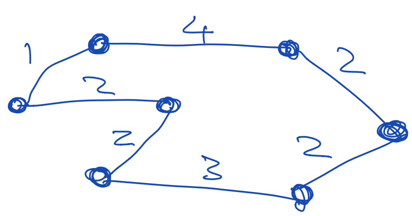


Step 3:

Find Eulerian (EG) Cycle while Taking Short-Cuts



(A)



Claim: (A) is a 2-approx. Sol. for MTSP

✓ $O \geq |MST|$

✓ $|EG| = 2|MST|$

$A \leq |EC| \leq |EG| = 2|MST| \leq 2O$

↳ Eulerian Circuit

$\Rightarrow \boxed{A/O \leq 2}$

1.5-approx. Algo. for MTSP

Claim*: The number of odd-degree Nodes is even

* After Constructing the MST

* $\overbrace{\sum_{\forall v_i \in V} d(v_i)}^{X_1}$ is Even

$= 2|E|$

bcz every edge is counted for exactly 2 nodes

* $\overbrace{\sum_{\forall \text{ Even-degree nodes}} d(v_i)}^{X_2}$ is Even

obvious

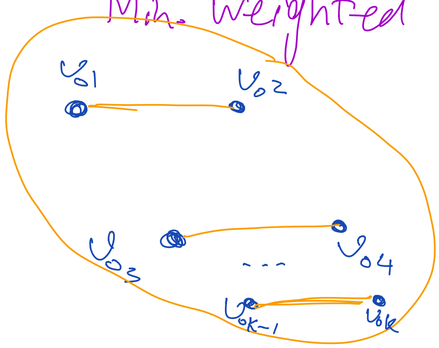
$\underbrace{X_1}_{\text{even}} = \underbrace{X_2}_{\text{even}} + \underbrace{\sum_{\forall \text{ odd-degree nodes}} d(v_i)}_{\text{even}}$

$\Rightarrow \sum_{\forall \text{ odd-degree nodes}} d(v_i)$ is Even

$\Rightarrow$ There are even number of odd-degree nodes in MST

Step 1: Find MST

Step 2:
identify the odd-degree nodes
and Find the
Min. weighted Matching



Step 3:
add all of these edges to MST

⇒ The result is an Eularian Graph

Step 4:
Follow the Eularian Cycle while Taking ShortCuts