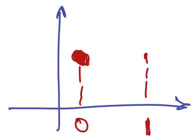
Repeated Sampling

e.g. Female to Male Ratio in a Society

1st Sample



underestimating

Using n Samples

$E[X] = \mu = 0.5$ → The mean of the rand. variable

$Stdev[X] = \frac{\sigma}{\sqrt{n}}$

Unbiased Sampling

how to generate Unbiased Samples based on an underlying distribution

Sampling

① Uniform between $[a, b]$

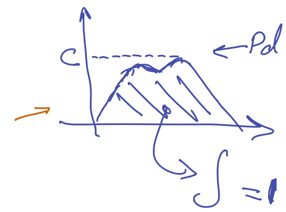
- Psuedo Random Generators
  - → $U[a,b]$: returns a real number range $[a,b]$

③ Sampling from a known distribution

- 2-1 Inverse CDF
- 2-2 Monte Carlo Rand. Gen.

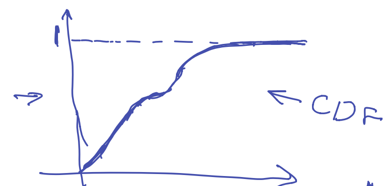
Inverse CDF Method

e.g.

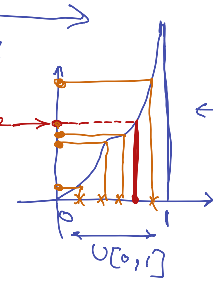


CDF: Cumulative Density function

$$F(X) = \int_{-\infty}^{X} P_x dx = P(x \leq X)$$



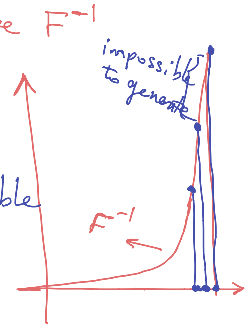
Compute F^{-1}



- F = CDF of the distribution
- F^{-1} = Compute the inverse of F
- for $i = 1$ to n → # Samples
  - $u = U[0,1]$
  - $S_i = F^{-1}(u)$

→ requirement: You should be able to compute F^{-1}

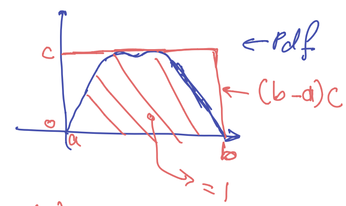
→ Because of digital #s, large ranges in tail may be impossible to generate



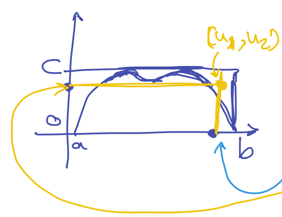
Monte-Carlo Random Generator

→ Accept-Reject Method

e.g.



$P(\text{inside}) = \frac{1}{c(b-a)}$



- $u_1 = U[a, b]$
- $u_2 = U[0, c]$
- accept the sample $\boxed{u_1}$ if (u_1, u_2) is below the curve of pdf // if $u_2 \leq f(u_1)$

otherwise Try again

→ Adv. : It works for any odd-shape distribution

→ disadvantage :

Depending on the shape of distribution it may reject many samples

→ Poor Performance (Slow)