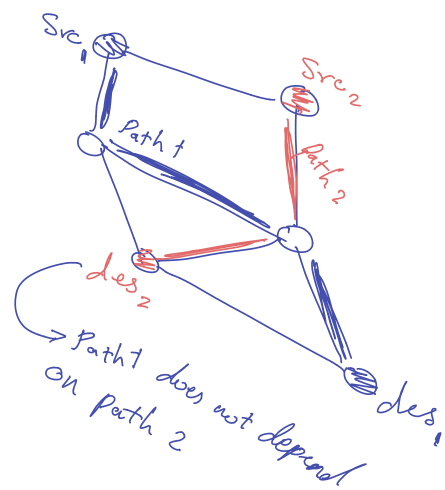
Oblivious Routing

assume a network with N nodes

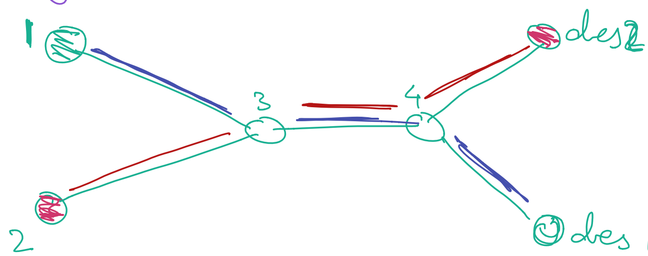
a Routing is oblivious, if selection of the routes do not depend on the other routes



Assumptions:

1. Packets are sent through edges in a Synchronous manner
   ↳ at every step 1 packet can be sent through an edge
2. at every step, an edge can carry at most 1 message
3. every node, for each of its edges has a queue

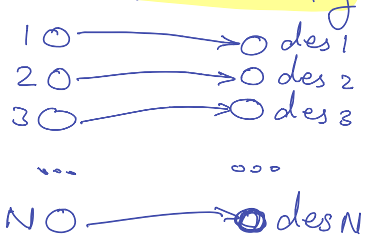
e.g.



Step 1	Step 2	Step 3	Step 4
(1): 1 → 3	(1): 3 → 4	(1): 4 → des 1	(2): 4 → des 2
(2): 2 → 3	(2): wait	(2): 3 → 4	

⟵ 4 steps ⟶

Permutation Routing



* every nodes sends exactly one message
* " " Receives " " "

Theorem

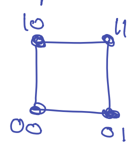
For any deterministic (oblivious) Routing Strategy, there is a Permutation (Bad Case) for which Routing requires $\Omega\left(\sqrt{\frac{N}{d}}\right)$ Steps (d is the degree of Nodes)

Network: Binary HyperCube → $d = n$

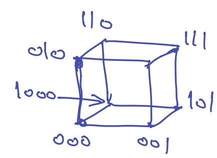
diameter $= n$

↳ max dist. b/w a pair of nodes

e.g.: 2bit



3bits



⇒ There are $N = 2^n$ nodes (n is # bits)

Two nodes are adjacent, if they are different in only one bit

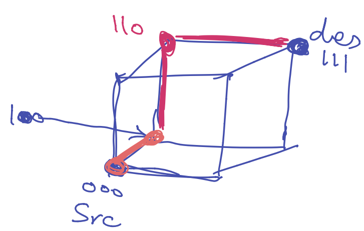
A deterministic Routing Algo. (Bit Fixing)

To move from a Src to dest.,
Start from the ID of the Src, from left to right
fix the bits to get to destination.

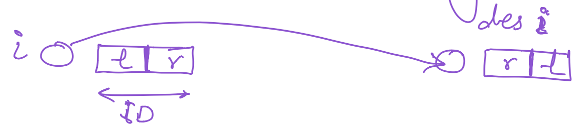
e.g.: Src. 01101100 ⟶ des. 01001111

	Step1	Step2	Step 3
Src	01001100	01001110	01001111
			des

e.g.2:

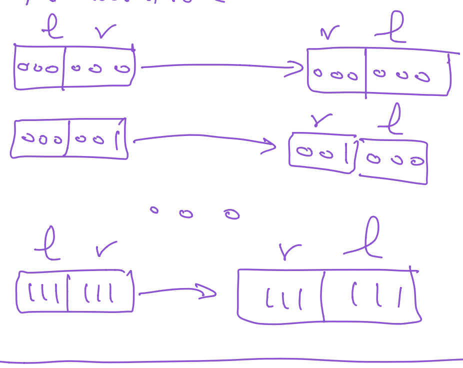


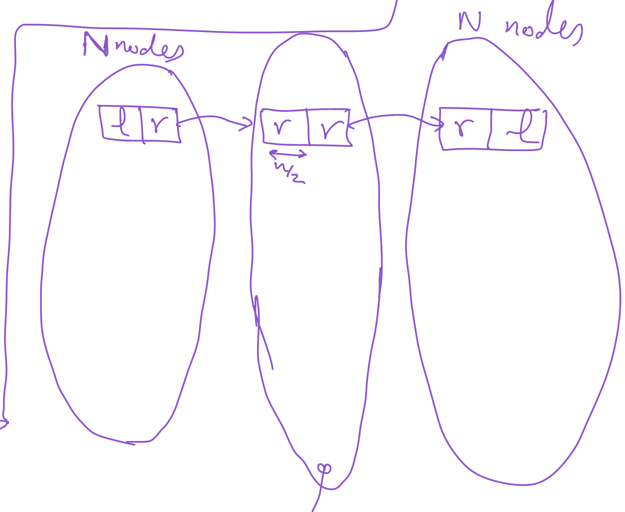
Bad Case for Bit Fixing



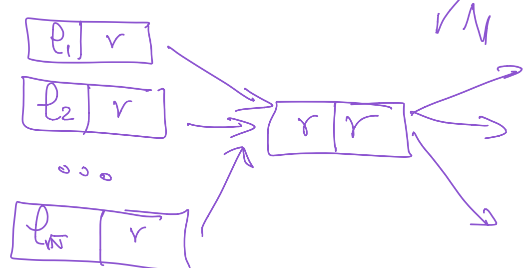
$\Omega\left(\sqrt{\frac{N}{n}}\right)$

Permutation



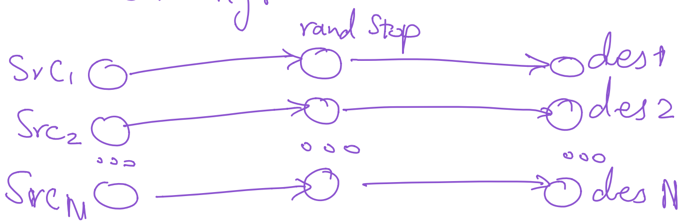


$\to 2^{n/2} = \sqrt{N}$



$\hookrightarrow \Omega\left(\sqrt{\frac{N}{n}}\right)$

Randomized Alg.



from a Source, first go to a random (indep.) Stop,
from the Stop, go to destination.

↳ Routing: Bit Fixing

Analysis: for a specific message, expect # of steps to get to dest.

- we consider Src ⟶ Stop.

① $E[|Path|] = n/2$

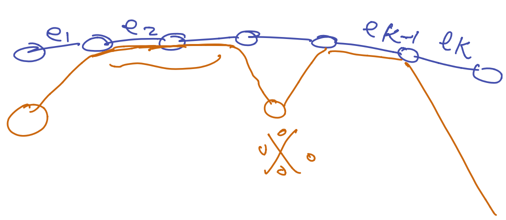
② # Steps = |path| + # delay

$E[\#\ Stepts] = n/2 + \#\ delay$?

- let $\langle e_1, e_2, \dots, e_k \rangle$ be the path (following Bit Fixing)

- The delay is bounded by $\underline{\#\ Paths}$ that share any of $e_1 \dots e_k$
S

Obs.



Expect # of Paths that share an edge $\boxed{e_i}$

$E[T(e_i)] = ?$

(1) # of Paths $= N$, $E[|P_j|] = n/2$

$E[\text{Sum of edges in all paths}] = \frac{Nn}{2}$

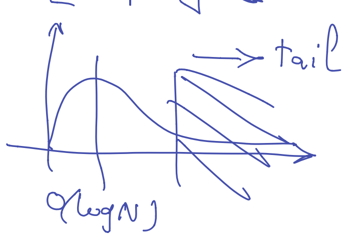
(2) # of edges $= \frac{Nn}{2}$

(1), (2) $\Rightarrow E[T(e_i)] = \frac{Nn/2}{Nn/2} = \boxed{1}$ ✓

$\Rightarrow E[\#\text{delay}_j] = E\left[\sum_{i=1}^{k} T(e_i)\right] \leq \sum_{i=1}^{n} E[T(e_i)] = \boxed{n}$ ✓

$E[\text{Total delay Src} \rightarrow \text{des}] = 2n$

$E[\#\text{Steps}] \leq n + 2n = 3n = O(\log N)$



$E[\#\text{delay}] = E\left(\sum T(e_i)\right)$

→ Sum of indep Rand. variables ?

→ use Chernoff Bound.