3D-Matching

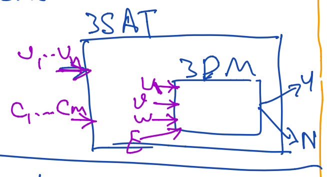
Given a 3partite Graph: U, V, W,
is there a perfect Matching

3DM $\in$ NP-Complete

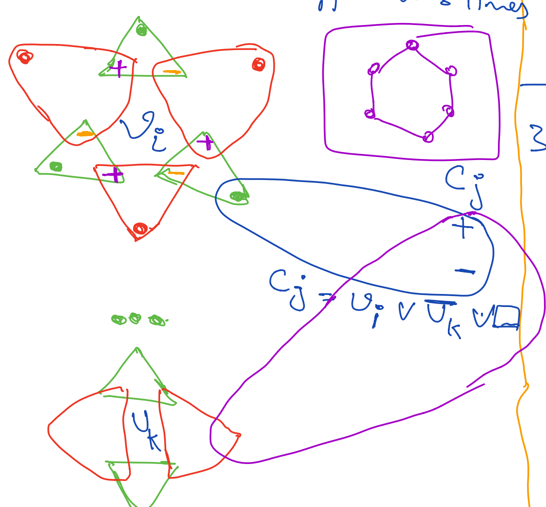
Step 1: 3DM $\in$ NP ✓

Step 2: Reduction

3SAT $\leq_p$ 3DM



assume that v_i has appeared 3 Times



Exact Cover by 3-Sets (X_3C)

a universe of items $U = \{I_1, \dots, I_n\}$

Collections of sets

$$\mathcal{S} = \{S_1, S_2, \dots, S_m\}$$

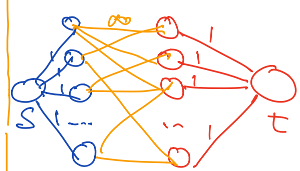
$\forall S_i : |S_i| = 3$

$\bigcup S_i = \mathcal{S}$

Obj: Select min Sets that Cover all items

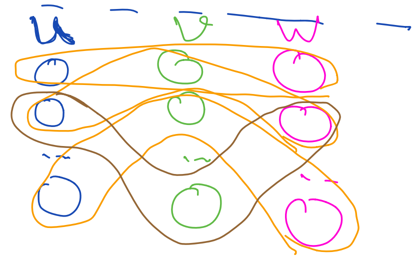
$$\boxed{X_3C \in \text{NP\_Complete}}$$

2DM

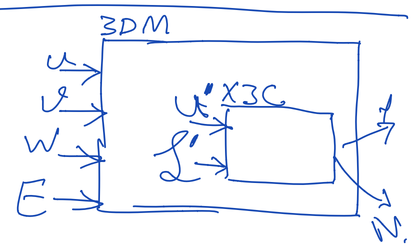


Polynomial

3DM



3DM $\leq_p X_3C$

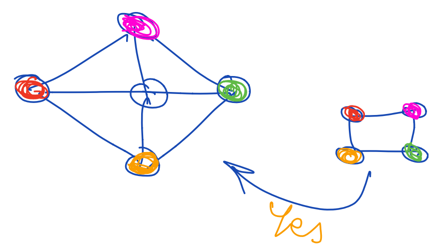


$$U' = U \cup V \cup W$$

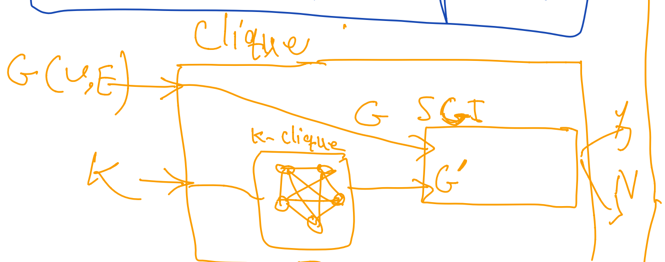
$$S' = E$$

Subgraph Isomorphism

Given a graph $G(V,E)$ and another graph $G'(V',E')$ is there an induced subgraph G_{sub} of G, s.t. there is matching b/w nodes/edges of G' and G_{sub}



$\boxed{\text{SubGI} \in \text{NP-Complete.}}$



Graph Isomorphism

given $G(V,E)$ and $G'(V',E')$ are G and G' isomorphism

GI ∈ NP ✓

given a certificate (matching of nodes), it is easy to verify it ← $O(n+m)$

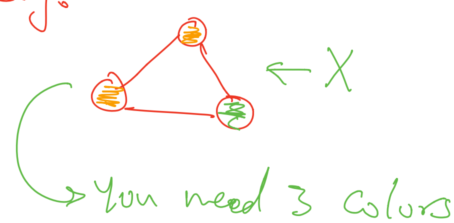
$\boxed{\text{GI} \overset{?}{\in} \text{NP-Complete?} \quad \text{UNKNOWN}}$

Graph Coloring

Given a graph G, Find min # of Colors*, s.t. no two adj. nodes get the same color

* Colors are assigned to nodes, not edges.

e.g.



Q_1: What graphs are 1-Colorable

Graphs w/t no edges

Q_2: Is deciding if g is 2-Colorable in P.

Yes. Check if it bi-partite

Q_3: if there is a clique of size k, you need at least K colors.

$\boxed{\text{3-Coloring} \in \text{NP-Complete}}$