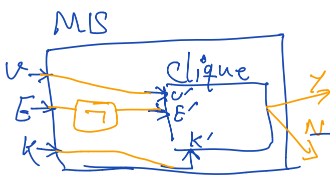
Clique ∈ NP-Complete

Step 1: Clique ∈ NP

given a certificate (a set of nodes & edges)
check if nodes & edges belong to G
and if it is a complete graph of
size k. ← $O(k^2)$ ✓

Step 2: Reduction

MIS $\leq_p$ Clique



$v' = v$

$\forall (u,v) \notin E$, add (u,v) to E'

$k' = k$

✓

Set-Cover

A universe of items $U = \{I_1, I_2, \dots, I_n\}$
and
a collection of sets

$S = \{S_1, S_2, \dots, S_m\}$

$S_i \subseteq U$

$\bigcup_{\forall S_i \in S} S_i = U$

Union

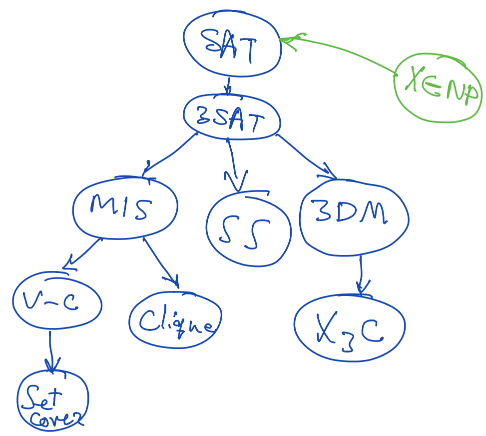
Objective:

find min # of sets that cover
all items.

Set-Cover ∈ NP-Complete

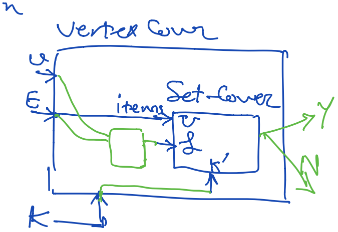
Step 1: Set-Cover ∈ NP

given a collection of sets (certificate)
it is easy to check if all
items are covered ← $O(nm)$



Step 2: Reduction

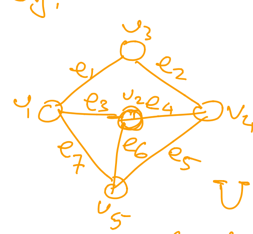
V-C $\leq_p$ S-C



$U = E$

$\forall$ vertex $v_i \in V$, add a set to S
and add all edges connected to v_i
to S_i.

e.g.



$U = \{e_1, e_2, \dots, e_7\}$

$S = \{\{e_1, e_3, e_7\},$
$\{e_3, e_4, e_6\},$
$\{e_1, e_2\}, \dots\}$

V-C has a cover of size k
if S-C has one " " k

✓

Subset Sum:

Given a set of numbers $I = \{I_1, I_2, \dots, I_n\}$ and a target t

is there a set $S \subseteq I$

$$\sum_{\forall I_i \in S} I_i = t$$

e.g.

5	9	1	3	8	11	21	7

I

$t = 15$

$\Rightarrow$ Yes: $5 + 9 + 1 = 15$

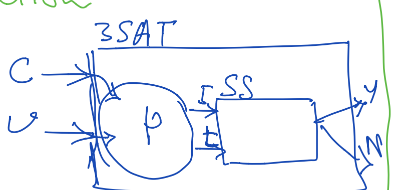
SS $\in$ NP-Complete

Step 1: SS $\in$ NP

given a set of numbers check if those add up to t $\leftarrow O(n)$ ✓

Step 2: Reduction

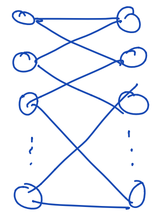
$C_1: (\bar{v}_1 \vee v_3 \vee v_n)$



	v_1	...	v_n	c_1	c_2	...	c_m
v_1	1	...	0	0		...	
$\bar{v}_1$	1		0	1			
...	0	...	⋮	...	...		
v_n	0		1	1			
$\bar{v}_n$	0		1	0			
C_{11}	0		0	1			
C_{12}	0	...	0	2	0	...	0
C_{21}	0	0	0	0	1	...	0
C_{22}	0	0	0	0	2	...	0
...	0	–	..	0	0	...	0
0	0	...	.	0	0	...	0
0	0	...	.	0	0		0
C_{m1}		...	-	0	0	...	1
C_{m2}	0	...	.	0	0	...	2

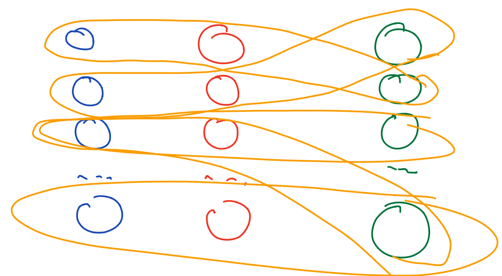
t	1	...	1	3	3	...	3

2D-Matching (Matching)



$\rightarrow$ If there is a matching

3DM



Given a 3partite Graph, is there a perfect matching

3DM $\in$ NP-Complete.