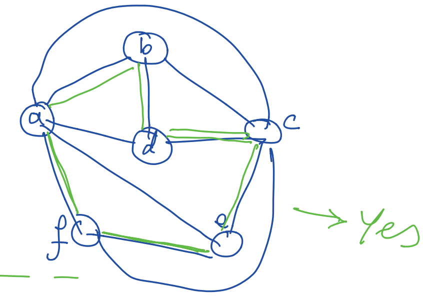
Hamiltonian Cycle (H-C)

Given a graph $G(V,E)$

Is there a Simple Cycle that passes through all nodes

e.g.



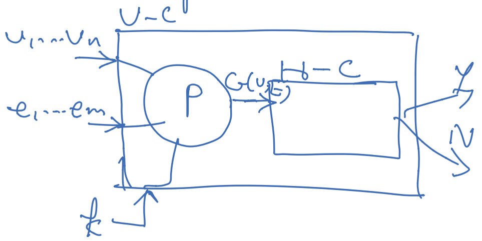
$H-C \in NP$-Complete

Step 1: $H-C \in NP$

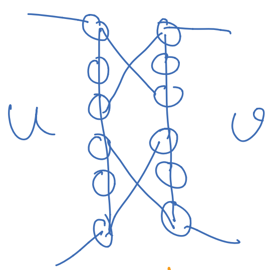
given a Certificate (a sequence of nodes), if it is a H-C ← $O(n)$

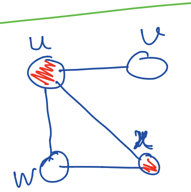
Step 2: Reduction

$V\text{-}C \leq_p H\text{-}C$



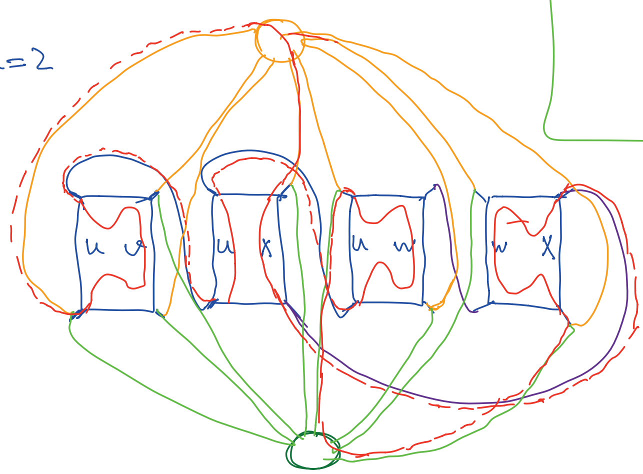
$\forall (u,v) \in E$

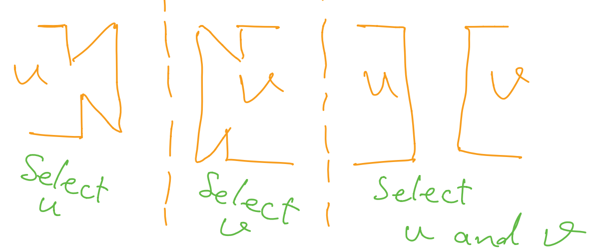




V-C of 2

K=2





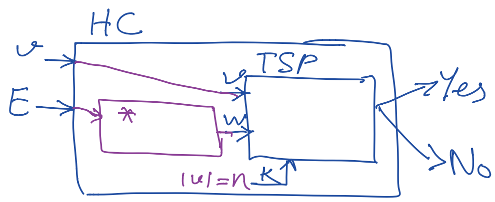
Traveling Salesman Problem (TSP)

Given a Complete weighted Graph
Find a Simple Cycle that passes
Through all nodes and has the min cost

TSP ∈ NP-Complete

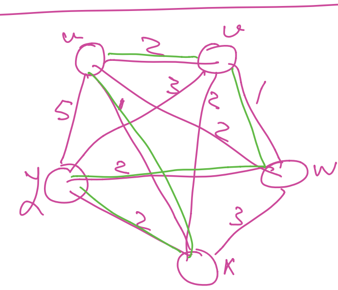
(A) TSP ∈ NP ✓

(B) Reduction



$* \ \forall (u,v) \in E$
Set $w[u,v] = 1$
$\forall (u,v) \notin E$
Set $w[u,v] = \infty$

e.g.



is there a complete cycle of Cost at most 10? Yes

Set Partitioning (SP)

Given a Set of Numbers U
is there a partition U
to two Sets, U_1, U_2
Such that

$$\sum_{I \in U_1} I = \sum_{I' \in U_2} I'$$

e.g.

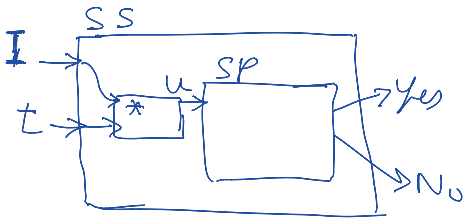
U: | 5 | 2 | 4 | 6 | 3 | 1 | 2 | 1 |

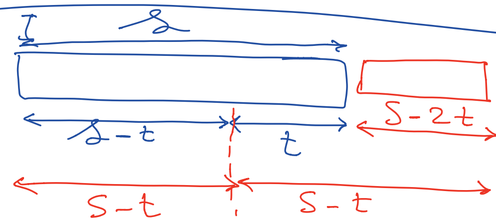
U_1: 4, 2, 5, 1
U_2: 6, 3, 2, 1
Yes

Ⓐ $SP \in NP$: Given u_1 & u_2, it is easy to check if it is a valid solution ← $O(n)$

Ⓑ Reduction

$SS \leq_p SP$





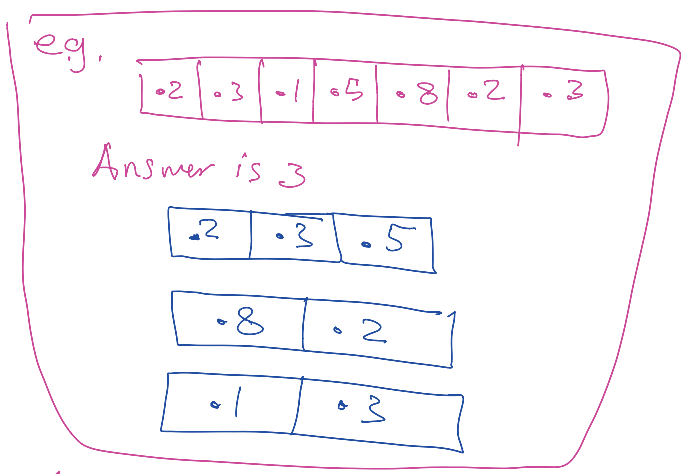
$$* \; u = I \cup \left\{ \sum_{I_i \in I} I_i - 2t \right\}$$

Bin Packing

Given a set of numbers each $\in (0, 1]$
what is the min # of bins each with capacity 1
to pack all the numbers

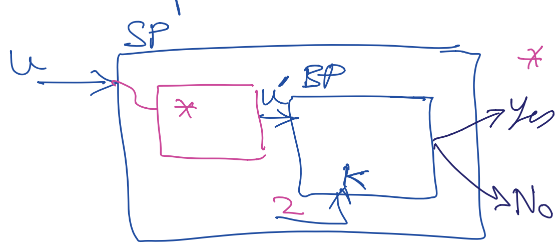
Bin Packing (BP) ∈ NP_Complete

Ⓐ $BP \in NP$ ✓ $O(n)$



Ⓑ Reduction

$SP \leq_p BP$



$* \; \forall u_i \in u$

add $2u_i / \sum_{\forall u_j \in u} u_j$ to u'

0/1 Knapsack

Given a set of items, each with profit P_i and weight W_i, and a backpack of capacity C

what is the max profit of selecting items

s.t.

$$\sum_{\forall u_i \in S} w_i \leq C$$

DP Solution

$O(nC)$

Knapsack ∈ NP-Complete

(A) Knapsack ∈ NP ✓ ← $O(n)$

(B) Reduction

$SS \leq_p$ Knapsack