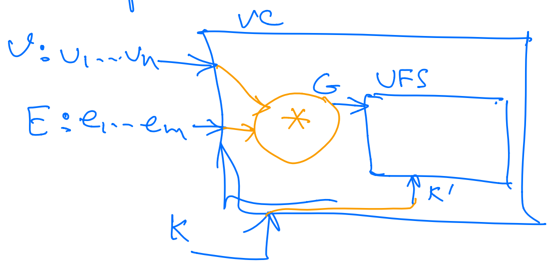
Exam 1 - Solution

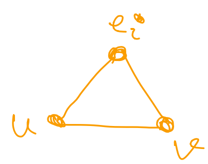
[Undirected Feedback Set Problem]

VC $\leq_p$ UFS

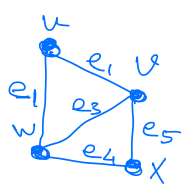


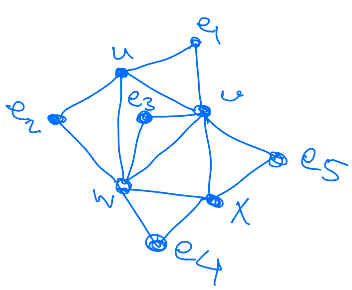
* $\forall (u,v) \in E$

✓ add



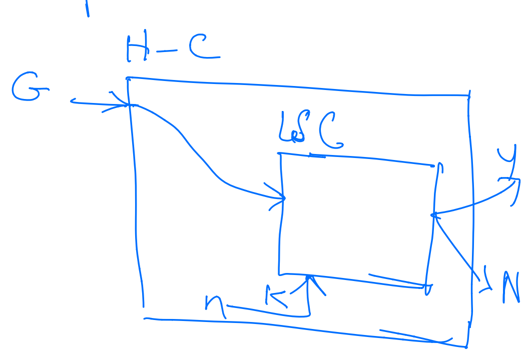
e.g.





[Longest Simple Cycle]

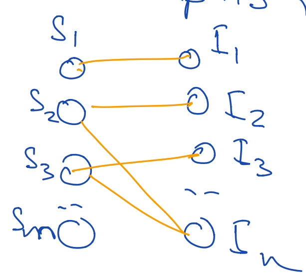
H-C $\leq_p$ LSC

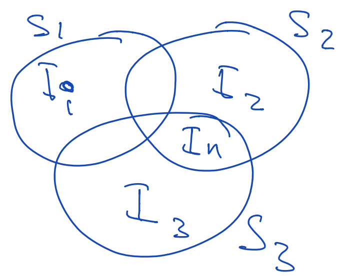


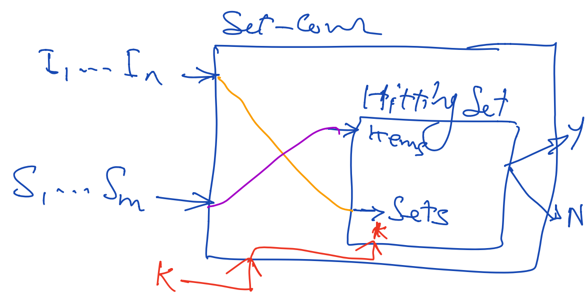
[Hitting Set Problem]

V-C $\leq_p$ HS

Set Cover $\leq_p$ HS ✓



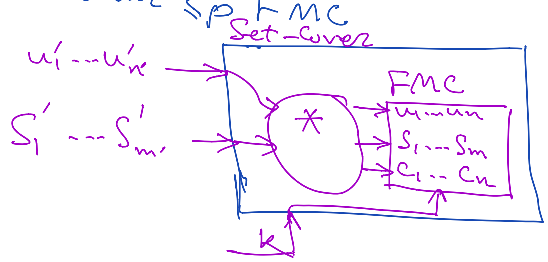




Fair Max-Cover

Set_Cover $\leq_p$ FMC

Max K-Cover $\leq_p$ FMC



* $\forall u_i$, add u_{i1}, u_{i0}

$C_{i1} = 1$, $C_{i0} = 0$

$\forall S_j'$ where $u_i \in S_j'$, add u_{i1} and u_{i0} to S_j

Approximation Algorithms

Assume O is the optimal value for a problem X

Assume an algorithm finds another Solution with value A

A is an approximation Algorithm for X if it Satisfies a bounded approximation-Ratio

?

X is max. (e.g. MIS) $\quad O/A$ is the approx.-ratio

X is min (e.g. V-C) $\quad A/O$ " " "

$\Rightarrow$ Approx-ratio ≥ 1 $\quad \Rightarrow$ for optimal: approx-ratio $= 1$

We want to design alg. with min approx. ratios

* Group1: There exists an approx. alg. with fixed approx. ratio
  ↳ V-C
* Group2: It is not possible to design an alg. with bounded approx. ratio
  ↳ Clique
* Group3: for a user provided value $\epsilon > 0$; we can Satisfy approx-ratio of $(1 + \epsilon)$

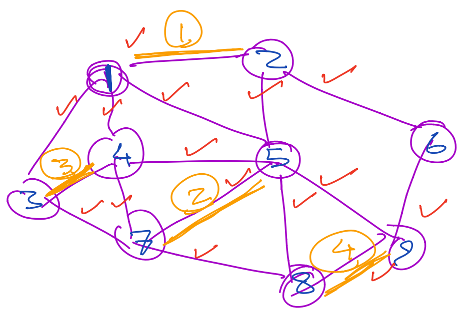
An Approx. Alg. for Vertex Cover

$S = \{\ \}$

while an uncovered edge left

- Select an " edge $(u, v) \in E$
- add u and v to S
- mark all edges that are connected to u OR v as covered

return S



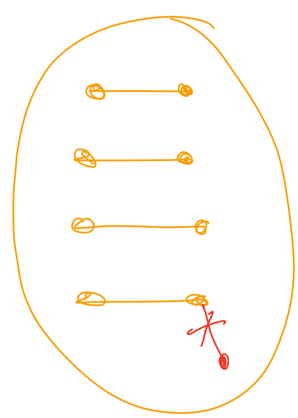
1. $S = \{1, 2\}$
2. $S = \{1, 2, 5, 7\}$
3. $S = \{1, 2, 5, 7, 3, 4\}$
4. $S = \{1, 2, 5, 7, 3, 4, 8, 9\}$

This Alg. has approx. Ratio of 2

Observation: All of the selected edges are disjoint

the optimal Solution should at least Cover these edges

↳ for each of these edges it should select at least one vertex



$$\Rightarrow O \geq |\text{Selected edges}|$$

$$A = |S| = 2\,|\text{Selected edges}|$$

$$\leq 2O$$

$$\Rightarrow A/O \leq 2$$

MIS

if optimal solution V-C selects k nodes

" " MIS has $(n-k)$ nodes

Using the approx. Alg. for V-C : $k' \leq 2k$

	V-C	MIS
O	k	$n-k$
A	$k' \leq 2k$	$n-k'$

MIS:

$$\frac{(n-k)}{(n-k')}$$

Unbounded

Assume: $k = n/2$

$k' = n$

$$\text{MIS} \quad \frac{(n - n/2)}{(n-n)} = \infty$$