V-C:
$\{v_1 \cdots v_n\}$
$\{e_1 \cdots e_m\} = E$
find $S \subseteq V$, s.t. all edges are hit
$|S|$ is minimized

$$\min \sum_{i=1}^{n} x_i \quad // \; x_i \text{ indicates if } v_i \text{ is selected}$$

s.t.
$\forall (v_i, v_j) \in E$
$x_i + x_j \geq 1$
$\forall x_i \in \{0, 1\}$

Relax to LP

$$\min \sum_{i=1}^{n} x_i$$

s.t.
$\forall (v_i, v_j) \in E$:
$x_i + x_j \geq 1$

$\forall x_i \geq 0$

Solve LP

let X^* be the opt solution for LP

Rounding:
$\forall 1 \leq i \leq n$

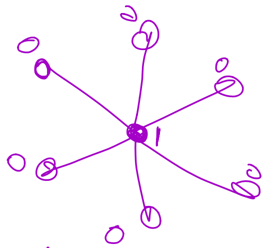
$$X^+ = \begin{cases} 1 & \text{if } x_i^* \geq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

(A) X^+ is a valid V-C

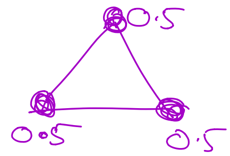
Because
$\forall (v_i, v_j) \in E$:
at least one of x_i^* or x_j^* should be $\geq 1/2$

(B) X^+ is a 2-approx. solution for V-C

$$\text{opt} \geq \sum x_i^*$$
$$= \sum_{\forall x_i < 1/2} x_i + \sum_{\forall x_j \geq 1/2} x_j$$
$$\geq \sum_{\forall x_j \geq 1/2} x_j$$
$$\geq \sum 1/2 = 1/2 \, \|X^+\|$$
$$= 1/2 \text{ approx} \Rightarrow \frac{\text{approx}}{\text{opt}} \leq 2$$



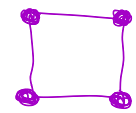
opt of LP is the opt of IP



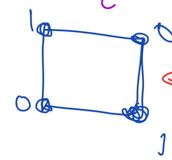
opt: Select 2

approx: Select <u>all</u>

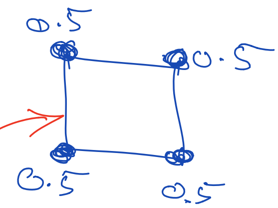
$\frac{3}{2}$: approx-ration



min $\sum x_i$



← Select 2 vertices



Select ALL vertices

approx-ratio: 2

weighted v-c

min $\sum w_i x_i$

s.t.

...

LP-relaxation is a 2-approx Solution for wv-c ?