Integer Programming

- $\{x_1, x_2, \dots, x_n\}$ x_i is integer
- m Linear inequality Constraints

e.g. $\sum a_{ji} x_i \leq b_j$

$$m \begin{cases} a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_j \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \leq b_m \end{cases}$$

$$A_{(m \times n)} X \leq b$$

- Opt function (min / max)
  ↳ linear
  $f(X)$

Linear Prog.

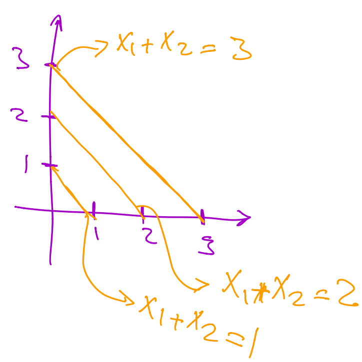
- $\{x_1, x_2, \dots, x_n\}$ x_i is a real number
- m linear inequality Constraints

$$AX \leq b$$

- Opt function (min / max)
  ↳ linear
  $f(X)$

Q_1: Is IP $\in$ NP-Complete? Yes

$$V\text{-}C \leq_p IP$$



Q_2: Is LP $\in$ NP-Complete?

e.g. : $\{x_1, x_2\}$

max $x_1 + x_2$

s.t.

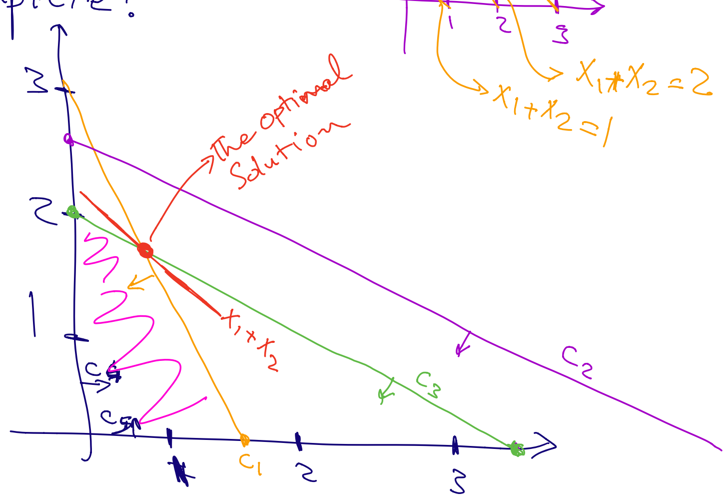
C_5: $x_1 \geq 0$

C_4: $x_2 \geq 0$

C_1: $2x_1 + x_2 \leq 3$

C_2: $x_1 + 2x_2 \leq 5$

C_3: $3x_1 + 5x_2 \leq 10$



Observation: - Solution Space is Convex

- (Max) opt function is a line we move towards origin & The first point it hits in the Sol. Space is the optimal Solution
- The optimal Solution is a <u>Corner point</u> in Sol. Space

if $n = 2$

there are $O(m^2)$ intersections

* Foreach intersection check if it is in the Sol. space

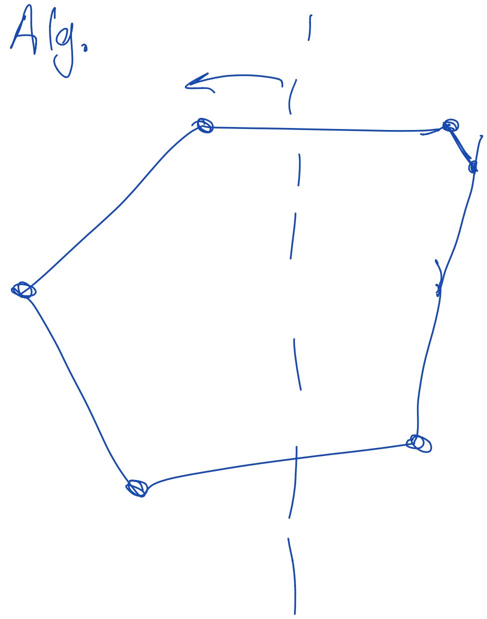
$O(m)$

* get the best

$O(m^3)$

In general? $O(m^n)$ ← exponential to <u>n</u>

Ellipsoidal Alg.



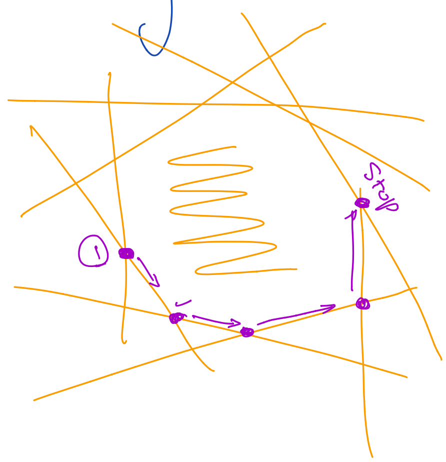
$O(n^c \log m)$

$O(\log(m^{O(n^c)}))$

$\in P$

Simplex Alg. (Exp. in worst Case, Polynomial in Average)

→ Hill-Climbing



max $x_1 + x_2$

* find a Corner point
* Find a neighboring corner
  → Drop one of the Equations (lines) and replace it with another one
* move to the new corner if it improves the objective value
* Stopping Condition:
  if none of the neighbors improve the function value