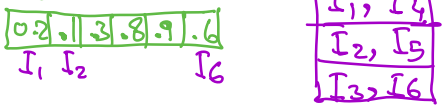
Bin Packing (BP)

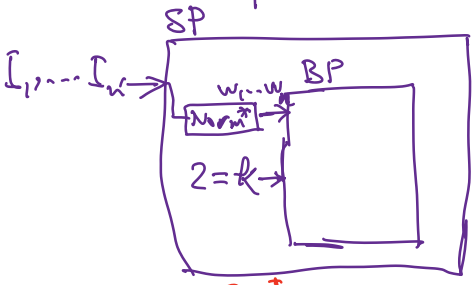
Given $U = \{w_1, \dots, w_n\}$, $0 \le w_i \le 1$, what is the min # bins each of max capacity 1 to pack all the elements.

e.g.:



BP $\in$ NP-Complete

1. BP $\in$ NP ✓
2. SP $\le_p$ BP



$$w_j = \frac{2 I_j}{\sum_{l=1}^{n} I_l}$$

0/1 knapsack

Given n Items, each with Profit P_i and weight w_i, and a backpack with capacity C, what is the set S of items to select s.t.

1. $\sum_{\forall I_j \in S} w_j \le C$
2. $\sum_{\forall I_j \in S} P_j$ is Maximized

↳ Dynamic Programing: $O(nC)$

$$z = \log C$$

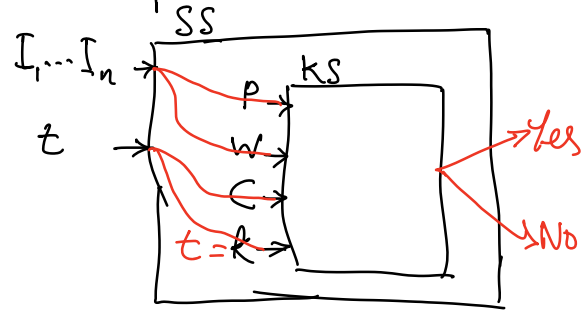
$$O(nC) = O(n 2^z)$$

↖ exponential to the Input Size

↳ Psedo Polynomial

0/1 knapsack (KS) $\in$ NP-Complete

1. KS $\in$ NP ✓
2. SS $\le_p$ KS



0/1 (Binary) Integer Programming

Given a Matrix $A_{n \times m}$ and a vector of binary variables $X_{m \times 1}$, and a vector of values b, find out if there exists an assignment to X such that

$$AX \le b$$

$$\begin{cases} x_1 + 3x_2 + 5x_3 \leq 2 \\ -2x_1 + 5x_2 + 4x_3 \leq 10 \\ x_1 - x_2 - x_3 \leq -5 \end{cases}$$

$$x_1, x_2, x_3 \in \{0, 1\}$$

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 5 & 4 \\ 1 & -1 & -1 \end{bmatrix}$$

$$X = \langle x_1, x_2, x_3 \rangle$$

$$b = \langle 2, 10, -5 \rangle$$

$$AX \leq b$$