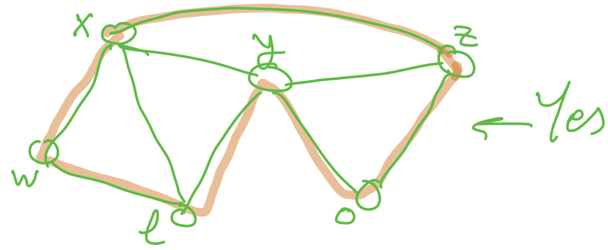
Hamiltonian Cycle (H-C)

Given $G(V,E)$, Find out if there exists a Simple Cycle that passes through all vertices in V

* Pass through every vertex exactly once

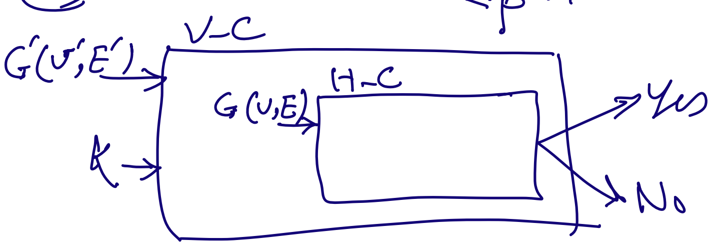
e.g.



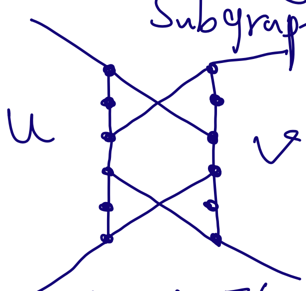
H-C ∈ NP-Complete

1. H-C ∈ NP: Given a chain of vertices as the certificate, checking if it is a Hamiltonian Cycle can be done in $O(|V|)$ ✓

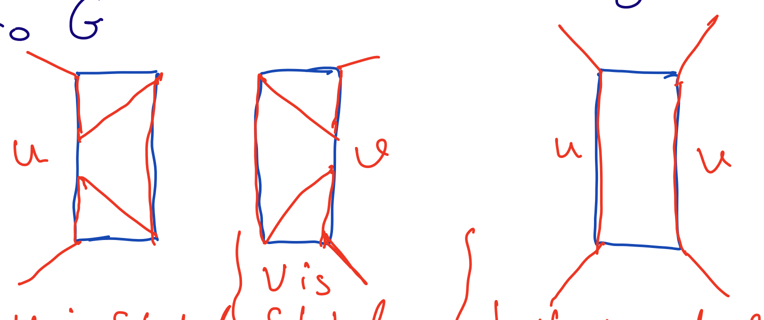
2. Reduction: $V\text{-}C \le_p H\text{-}C$



2-1. For every edge $(u,v) \in E'$, add the following subgraph to G

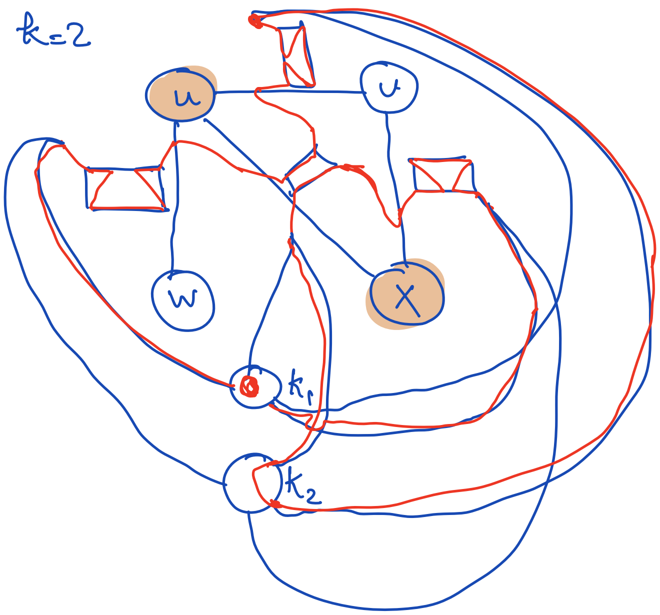


$(u,v) \in E'$

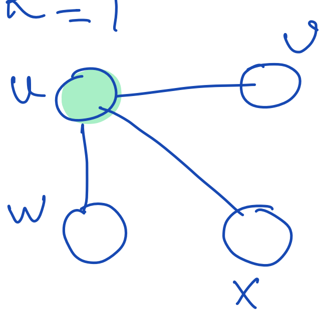


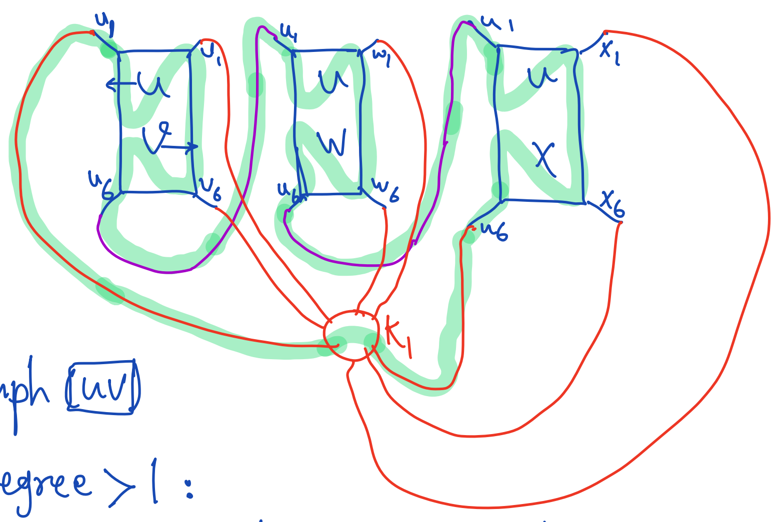
u is Selected | v is Selected | both u and v are Selected

$k=2$



$k=1$





① $\forall (u,v) \in E'$:
add the subgraph $\boxed{uv}$

② $\forall u \in V'$ with degree > 1:
Connect the gadgets corresponding to their edges:
$u_6 \longrightarrow u_1$

③ add k additional vertices & connect all free pins to all of them

$\Rightarrow$ the answer for $VC_k = 1$ if G has a H-C

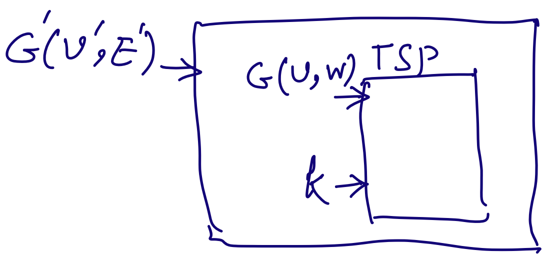
Traveling Salesman Problem (TSP)

Given a Complete weighted graph $G(V, w)$, find the Simple Cycle that passes through all vertices & has the min Cost.

$TSP \in NP\text{-Complete}$

1. $TSP \in NP$:
   Given $G(V, w)$, k, and a Certificate (a Simple Complete cycle), it is easy to Compute the weight and Check it against k ← $O(|V|)$ ✓

2. $H\text{-}C \leq_p TSP$



- $V = V'$
- $\forall (u,v) \in E'$: Set $w(u,v) = 1$
  $\forall (u,v) \notin E'$: Set $w(u,v) = |V|+1$
- $k = |V|$

Bin Packing (BP)

Given n elements $I_1, \ldots I_n$, where every element has a weight $w_i \in [0, 1]$, what is the min #bins (each with capacity 1) to Cover all elements.

$BP \in NP\text{-Complete}$