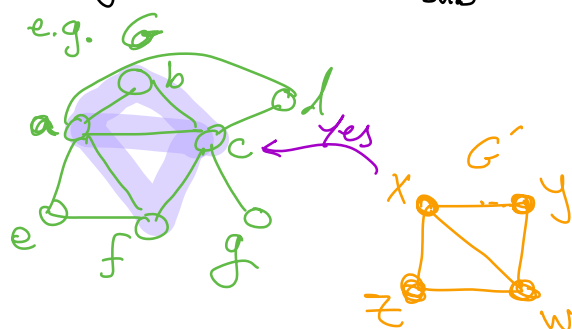Subgraph Isomorphism (SI)
Given a graph $G(V,E)$ and
a (smaller) graph $G'(V',E')$
Objective:
  find out if there exists an
  induced subgraph G_{sub},
  such that there is a 1-1
  mapping b/w the nodes & edges
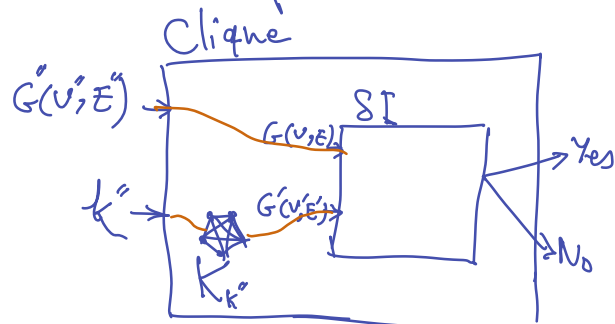 of G' and G_{sub}.

e.g. G

Yes

G'

- Subgraph Isomorphism ∈ NP-Complete

① SI ∈ NP.
  Given G and G' and a Certificate
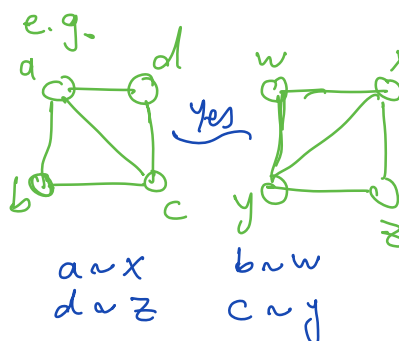 (an induced subgraph of G & the mapping)
  finding the 1-1 mapping can be
  done in $O(|V| + |E|)$

② Clique $\leq_p$ SI

Clique

$G''(V'',E'')$
$G(V,E)$
SI
k''
$G'(V',E')$
$K_{k''}$
Yes
No

Graph Isomorphism (GI)
Given $G(V,E)$ and $G'(V',E')$
find out if G is an isomorphism
for G'.

e.g.

Yes

a ∼ x   b ∼ w
d ∼ z   c ∼ y

GI ∈ NP-Complete ?

~~GI $\leq_p$ SI~~

UNKNOWN

Min Steiner Tree (MSteT)

Given a weighted Graph $G(V,E,w)$ and a set S of required vertices, find a Tree w/t min weight that includes all vertices in S

e.g.

$$3 + 2 + 2 + 2 = 9$$

MSteT $\in$ NP-Complete

① MSteT $\in$ NP

Given $G(V,E,w)$ and S, a value k, and a Certificate (a Tree), Check if Tree belongs to G, Covers all nodes in S, and Sum of weights is at most k.

$\longleftarrow$ $O(|V|)$

② VC $\leq_p$ MSteT

proof

$G(V',E')$ VC

Min Spanning Tree (MST)

Given a weighted Graph $G(V,E,w)$ find a Tree w/t min weight that includes all vertices

MST $\in P$

Proof

$k=2$

$$V = V' \cup E'$$

$$w_i = 1$$

$$S = E'$$

$$k = k' + |E'| - 1$$

$\forall e_j = (u,v) \in E':$
 add (u, e_j) and (v, e_j) to E
$\forall u, v \in V':$
 add (u,v) to E