Subset Sum

$U = \{I_1, I_2, \dots, I_n\}$

target value t,

goal

$\exists\ S \subseteq U$

$$\sum_{I_i \in S} I_i = t$$

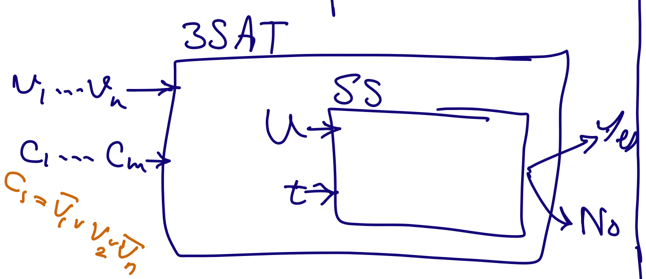
① $SS \in NP$

given a certificate $S \subseteq U$

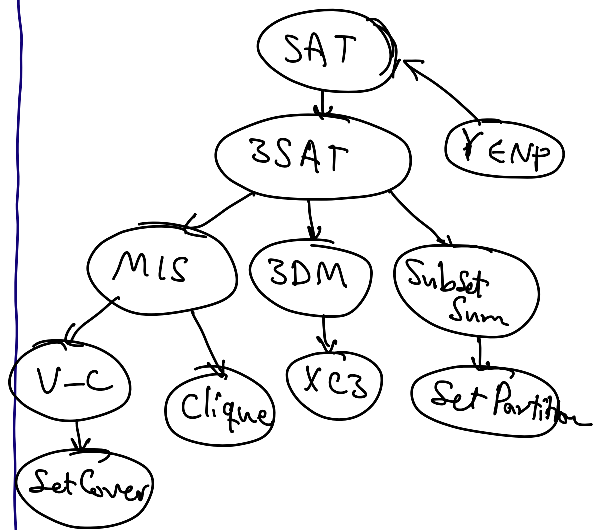
add up the values to see if it is equal to t.

$O(n)$ ✓

② $3SAT \leq_p SS$



		v_1	v_2	...	v_n	C_1	C_2	...	C_m
I_1	v_1	1	0	...	0	0		...	
I_2	$\overline{v_1}$	1	0		0	1			
I_3	v_2	0	1		0	1			
	$\overline{v_2}$	0	1		0	0			
	⋮	0	0		0	0			
	v_n	0	0		1	0			
	$\overline{v_n}$	0	0		1	1			
		⋮	⋮		⋮				
	c_{11}	0	0	...	0	1	0	...	0
	c_{12}	0	0	...	0	2	0	...	0
	c_{21}	0	0	...	0	0	1		
	c_{22}	0	0	...	0	0	2		
	⋮								
t		2	2	....	2	3	3	...	3



XC3: exact cover by 3-sets

Given $U = \{u_1, u_2, \dots, u_n\}$

a collection of sets $S_1 \dots S_m$

$S_i \subseteq U$ and $|S_i| = 3$

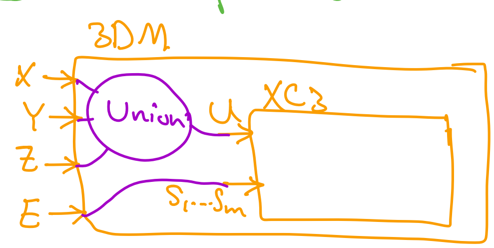
goal:

Find min # sets $\mathcal{S} \subseteq \{S_1 \dots S_m\}$

$$\bigcup_{S_i \in \mathcal{S}} S_i = U$$

① $XC3 \in NP$

② $3DM \leq_p XC_3$



Set Partition (SP) I_j is a number

given $\mathcal{U} = \{I_1 \dots I_n\}$

check if $\exists\, S \subseteq \mathcal{U}$ s.t.

$$\sum_{I_i \in S} I_i = \sum_{I_j \notin S} I_j$$

e.g.

$\mathcal{U} = \{1, 5, 8, 9, 4, 1\}$

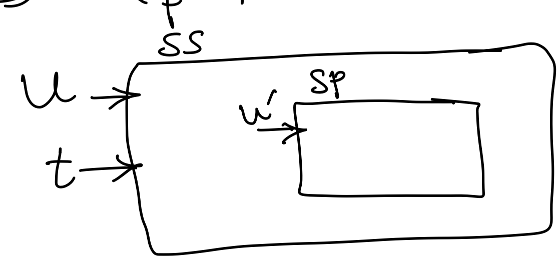
$S = \{8, 1, 4, 1\}$

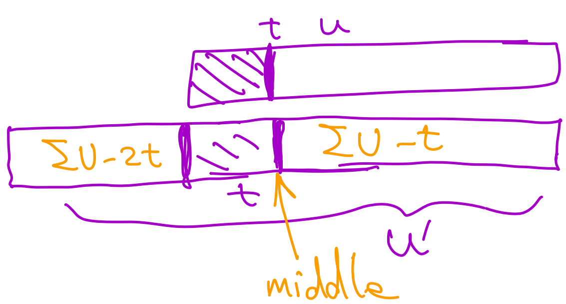
$\sum S = 14$

$\sum \bar{S} = 14$

① $SP \in NP$ ✓

② $SS \leq_p SP$





$\mathcal{U}' = \mathcal{U} \cup \{\Sigma U - 2t\}$

Max-Cover (MC):

Given a Universe of elements $\mathcal{U} = \{u_1 \dots u_n\}$, and a number k and $S_1 \dots S_m$ $\forall S_i \subseteq \mathcal{U}$;

Find k sets $\mathcal{S}$ $|\mathcal{S}| = k$ such that

$$\left| \bigcup_{S_i \in \mathcal{S}} S_i \right| \text{ is maximized}$$

① $MC \in NP$

③ $SC \leq_p MC$