$\mathcal{U} = \{u_1, \dots, u_n\}$

$S_1, \dots, S_m$

$S_i \subseteq \mathcal{U}$

$\bigcup_{\forall i=1}^{m} S_i = \mathcal{U}$

goal:

min # sets that cover all elements.

$S = \{S_{i_1}, \dots, S_{i_k}\}$

$\bigcup^{k} S_{i_k} = \mathcal{U}$

min k

S-C ∈ NP-Complete

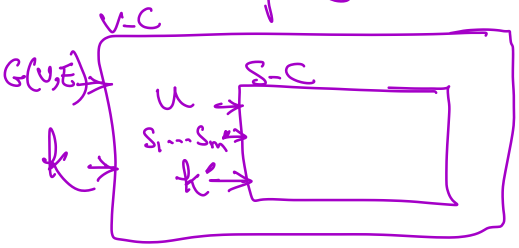
① S-C ∈ NP

given a certificate and $\mathcal{U}$, $S_1, \dots, S_m$, and a value k

take the union of sets in the certificate and check if it covers all the elements

✓ $O(n+m)$

② V-C $\leq_p$ S-C



$\forall e_i$ add e_i to $\mathcal{U}$

$\mathcal{U} = \{e_1 \dots e_m\}$

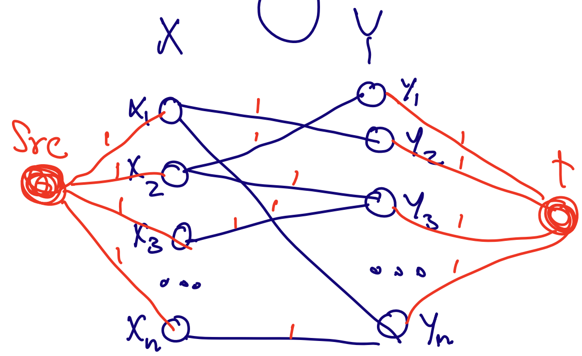
$\forall v_i$ add the set $\boxed{S_i}$

$S_i = \{e_j \mid e_j \text{ is incident to } v_i\}$

$k' = k$

A yes/No to Set Cover is a Y/N to Vertex Cover

2D Matching



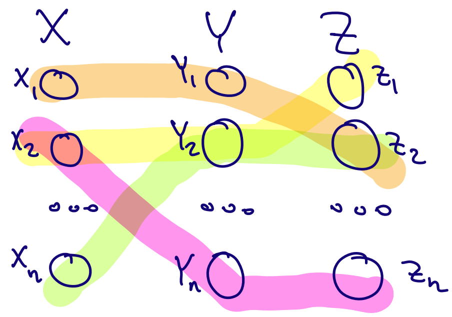
Given the sets $X = \{x_1 \dots x_n\}$ and $Y = \{y_1 \dots y_n\}$ and $E \subseteq X \times Y$

goal:

Is there a perfect matching $E' \subseteq E$, where every x_i and y_j is matched

$\Rightarrow$ 2DM ∈ P

3DM



Given $X = \{x_1 \dots x_n\}$
$Y = \{y_1 \dots y_n\}$
$Z = \{z_1 \dots z_n\}$
$E \subseteq X \times Y \times Z$

is there a Perfect Matching $E' \subseteq E$, s.t.

$\forall x_i \ \exists e \in E' \ x_i \in e$

and

$\forall y_i \ \exists$ " " $y_i \in e$

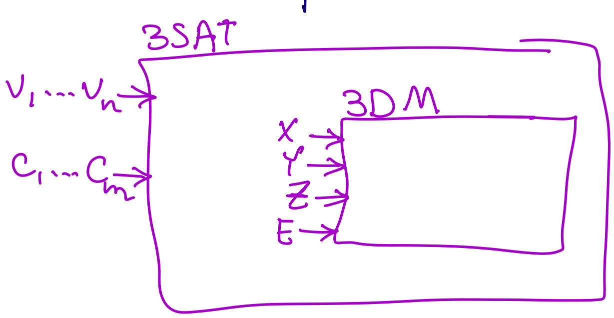
and

$\forall z_i \ \exists e \in E' \ z_i \in e$

---

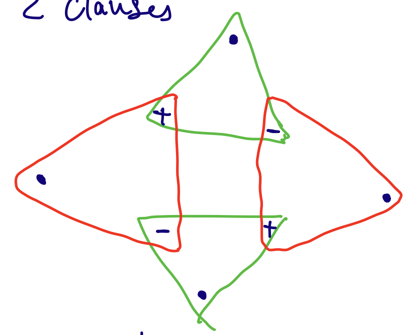
(1) 3DM $\in$ NP ?

(2) 3SAT $\leq_p$ 3DM

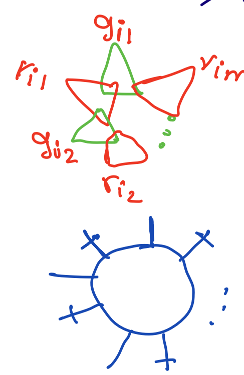


gadget for v_i

e.g. 2 clauses

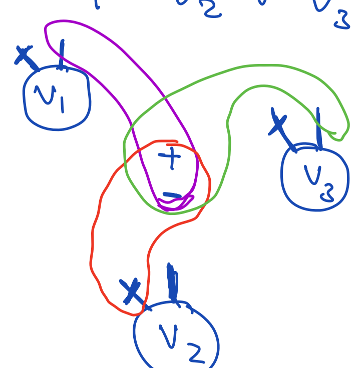


for m clauses $\rightarrow$ 2m edges



add 3 edges per clause C_j

$C_1 = v_1 \vee \overline{v_2} \vee v_3$



$C_2 = \overline{v_1} \vee \overline{v_2} \vee \overline{v_3}$

$\{1, 0, 1\}$

$\forall g_{ij}, r_{ij}$ add two nodes $\{+, -\}$ and create "cleaner" edges connected to $g_{ij}(\circ)$ and $r_{ij}(\circ)$

$\Rightarrow X = +$

$Y = -$

$Z = \circ$

$E = \begin{cases} \text{edges added for variables} \\ \text{" " " clauses} \\ \text{" " " cleaners} \end{cases}$

Subset Sum:

given a set of numbers $U = \{I_1 \dots I_n\}$ and a value t,

$U' \subseteq U$

s.t.

$$\sum_{\forall I_j \in U'} I_j = t$$