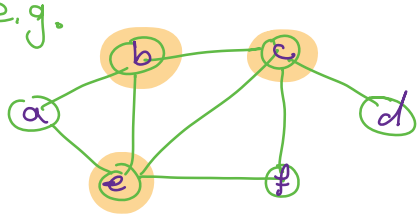
Vertex Cover (V-C):

Given $G(V,E)$, Find the min Set of nodes (S), Such that

$\forall (u,v) \in E$, $u \in S$ or $v \in S$

e.g.



- $\{a, b, c, d, e, f\}$ is V-C of Size 6.
- Opt = 3

  $\{b, c, e\}$

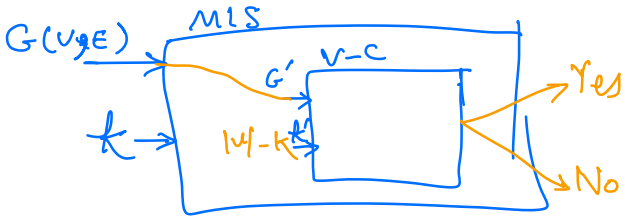
VC $\in$ NP-Complete

(1) V-C $\in$ NP:

Given $G(V,E)$, a value k, and a Set of vertices (S), Verify Every edge is connected to at least one node in S

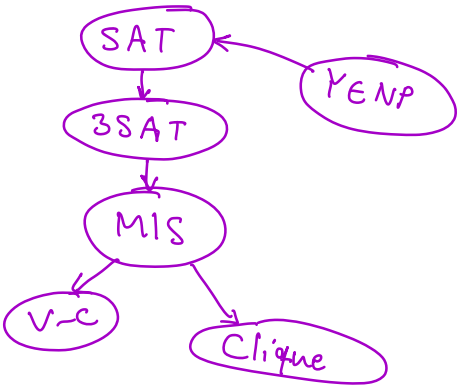
$\leftarrow O(|V|+|E|)$ ✓

(3) MIS $\leq_p$ V-C



$G'(V',E') = G(V,E)$

$k' = |V| - k$

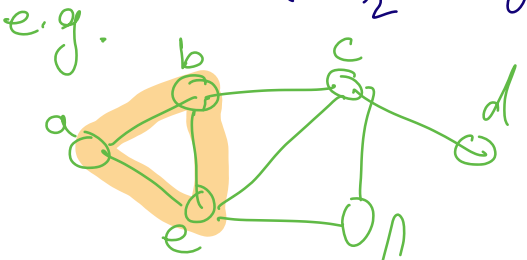


Clique:

Given $G(V,E)$, Find a Clique of Largest Size

A Clique of Size k is a Complete graph of k nodes ($\frac{k(k-1)}{2}$ edges)

e.g.



is $\{a, b, c\}$ a K_3? No ✗

is $\{a, b, e\}$ a K_3? Yes ✓

$\leftarrow$ opt = 3

Clique $\in$ NP-Complete

① Clique $\in$ NP:

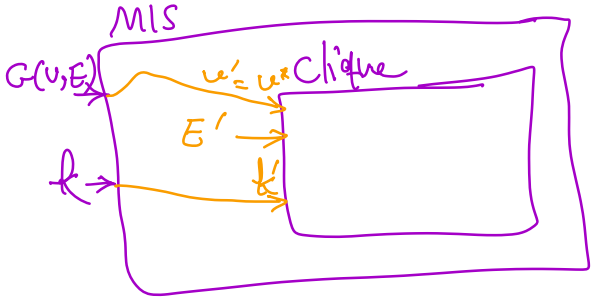
Given $G(V, E)$, an value k, and a set of nodes (S), verify (A) $|S| \geq k$

(B) $\forall u, v \in S, (u, v) \in E$

$\leftarrow O(|E|)$

② Reduction

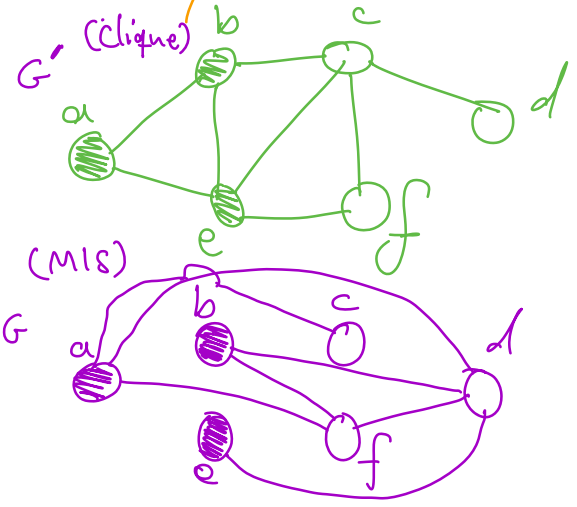
$MIS \leq_p$ Clique



$k' = k$

$v' = v$

$\forall (u, v) \notin E$, add $(u, v) \in E'$



Set-Cover:

Given a universe of elements

$U\{u_1, u_2, \ldots, u_n\}$

a Collection of sets

$S_1, S_2, \ldots, S_m$

$\forall S_i \subseteq U$

$\bigcup_{\forall i=1}^{m} S_i = U$

Select min # of Sets O

s.t.

$\bigcup_{\forall S_i \in O} S_i = U$

e.g.

$U = \{1, 2, 3, \ldots 10\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 3, 7, 8\}$

$S_3 = \{2, 4, 6, 8, 10\}$

$S_4 = \{1, 3, 5, 7, 9\}$

$\{S_1, S_3\}$ a Set Cover? X (No)

$\{S_3, S_4\}$ " " ? Yes

$\leftarrow opt = 2$