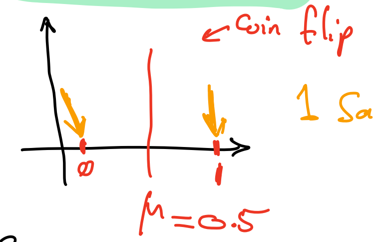
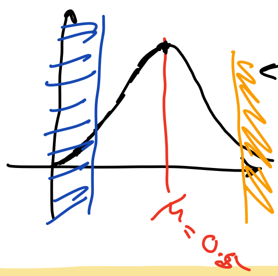
Chernoff Bound



1 Sample: { Underestimate / Overestimate

Repeat the process n times & Take the average



Centeral Limit theorem: Sum of indep. random variables follow the Normal Dist.

$$\mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$P(X \geq (1+\delta)\mu) \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu} \leq e^{-\delta^2 \mu / 3}$$

$\delta > 0$

$$P(X \leq (1-\delta)\mu) \leq e^{-\frac{\delta^2 \mu}{2}}$$

---

e.g. Suppose $P_h = 0.5$. What is the Prob. that after n flips of Coin at least 80% of them are head?

$$P(X \geq 0.8n) \leq \frac{0.5n}{0.8n} = \frac{0.5}{0.8} < 0.7$$

Using Markov.

$P(X \geq 0.8n)$, Using Chernoff bound

$(1+\delta)\mu = 0.8n$, $\mu = 0.5n$

$\Rightarrow (1+\delta)0.5n = 0.8n \Rightarrow \delta = 0.6$

$$P(X \geq (1+\delta)\mu) \leq e^{-\frac{\delta^2 \mu}{3}} = e^{\frac{-(0.6)^2\, 0.5n}{3}} = \frac{1}{e^{0.06n}}$$

e.g.: $n = 100$

$$\Rightarrow P(X \geq 80) \leq \frac{1}{e^6}$$

A football team wins every game w/p Prob. 0.75
what is the Prob. that in a Season of n games it wins in less than 0.5 of the games?

$$P(X \leq (1-\delta)\mu) \leq e^{-\delta^2 \mu/2}$$

$$\mu = 0.75n \ , \quad (1-\delta)\mu = 0.5n$$

$$\Rightarrow (1-\delta) = \frac{0.5}{0.75} \ , \quad \delta = 1/3$$

$$\Rightarrow P(X \leq 0.5n) \leq e^{-\frac{(1/3)^2 0.75n}{2}} = e^{-\frac{0.75n}{18}}$$