Random Number Generation

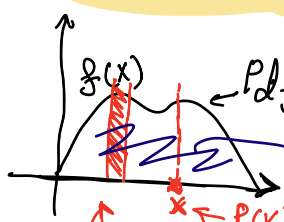
- how to generate UNBIASED samples from a given Prob. dist.?

- Assumption : $U[a,b]$ ← generates a random Uniform # in range $[a, b]$

- Inverse CDF

- Accept/Reject Monte-Carlo Sampling

Inverse CDF



← pdf = Prob. density function

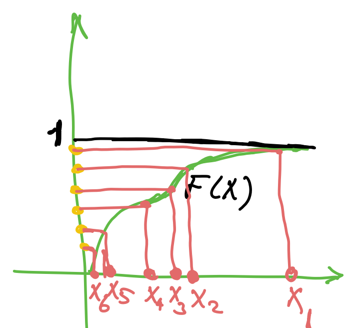
→ Area Under the Curve = 1

← $P(X) = \emptyset$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

Step 1 : Convert Pdf to Cdf

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

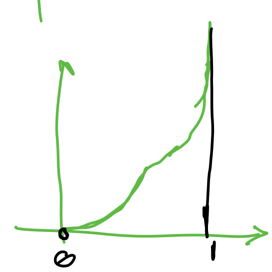


Step2 : Take The inverse of CDF

$F^{-1}(x) = \text{inverse}(F(X))$

Step 3 : $y = U[0, 1]$

return $F^{-1}(y)$

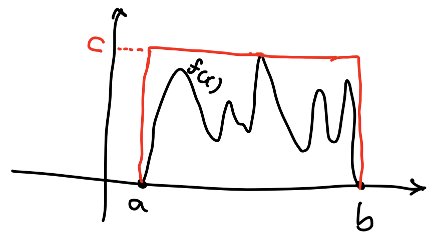


Draw Backs : (1) Needs to gen. F^{-1}

(2) Because of digital #s, large ranges in the tail may be impossible to generate

Monte-Carlo Random Generator

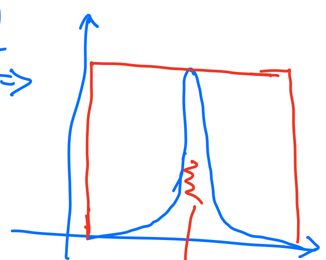
↳ Accept/Reject Method



$x_1 = U[a, b]$
$y_1 = [0, c]$ // $\langle x_1, y_1 \rangle$ is the "raindrop"
if ($f(x_1) \leqslant y_1$) // accept
    return x_1
else // reject (no Sample gets generated)
    Try again

Advantage: Can generate Samples from odd-Shape distributions
disadvantage: Depending on the Shape of distribution
Prob. of accept (generating Samples) may be
Small ⇒ Inefficient

Adversarial Example ⇒



$P_{accept} = \dfrac{\text{Area Under the Curve}}{c(b-a)} \rightarrow 0$

⇒ Keeps rejecting Samples

Summary

	Adv.	Disadv.
inverse CDF	Fast	- need F^{-1} - May not gen. wide ranges in tail
Monte Carlo	work for odd-Shape distributions	May be Inefficient