## Markov Inequality

useful when we only know the expected value

$$\boxed{P(X \geq t) \leq \frac{E[X]}{t}}$$

(No assumption on the distribution)

Proof.

$$E[X] = \int_{-\infty}^{\infty} x\, p_x\, dx$$

$$= \int_{-\infty}^{t} x\, p_x\, dx + \int_{t}^{\infty} x\, p_x\, dx$$

$$\geq \int_{t}^{\infty} x\, p_x\, dx$$

$$\geq \int_{t}^{\infty} t\, p_x\, dx = t \int_{t}^{\infty} p_x\, dx$$

$$\boxed{\int_{t}^{\infty} p_x\, dx = P(X \geq t)}$$

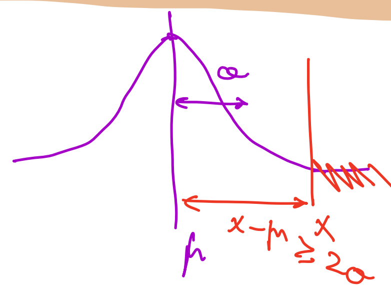
$$E[X] \geq t\, P(X \geq t)$$

$$\Rightarrow P(X \geq t) \leq \frac{E[X]}{t}$$

## Chebyshev Inequality

when we know the expected value and the variance

$$\boxed{P(|X - \mu| \geq t\sigma) \leq 1/t^2}$$




(No assumption on the data Distribution)

Proof.

$$Y = (X - \mu)^2$$

$$P(|X - \mu| \geq t\sigma) = P((X-\mu)^2 \geq t^2\sigma^2)$$

$$= P(Y \geq t^2\sigma^2)$$

Using Markov Ineq.

$$P(Y \geq \underbrace{t^2\sigma^2}_{T}) \leq \frac{E[Y]}{T}$$

$$= \frac{E[Y]}{t^2\sigma^2}$$

$$E[Y] = E[(X-\mu)^2] = \sigma^2$$

$$P(Y \geq t^2\sigma^2) \leq \frac{\sigma^2}{t^2\sigma^2} = 1/t^2$$