$P \subseteq NP$? Yes

- $P = NP$?
- $P \neq NP$?

} Don't Know

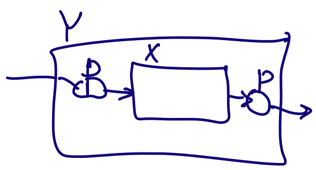
NP-Complete

$X \in NP$

and

$\forall\, Y \in NP$

$Y \leq_p X$



SAT (Satisfiability)

- Circuit SAT

Input: a set of binary variables $v_1, \dots v_n$

(Boolean) Clauses:

$\vee$ OR

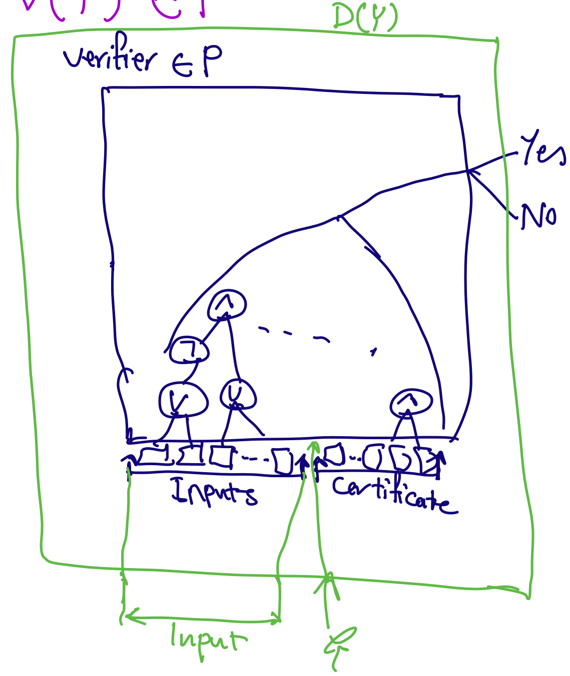
$\wedge$ AND

$\neg$ NOT

Given the binary variables and clauses forming a boolean circuit is there an assignment to $v_1 \dots v_n$ such that the output of expression is True (1)

$Y \in NP$

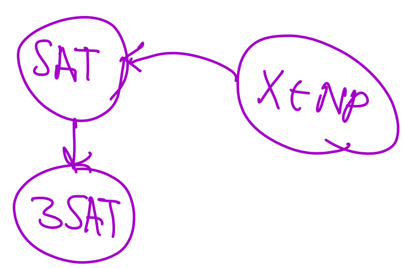
$V(Y) \in P$



$\Rightarrow$

SAT $\in$ NP-Complete

1. SAT $\in$ NP
2. $\forall\, Y \in NP$ $Y \leq_p X$



3SAT:

- Given $v_1 \dots v_n$ (Boolean Variables)
- A clause Contains exactly 3 variables + ($\vee$ $\neg$) operations
- Clauses are merged Using $\wedge$ (Boolean AND) operation

e.g.

X_1, X_2, X_3, X_4

$(X_2 \vee \neg X_3 \vee \neg X_4) \wedge$
$(X_1 \vee X_3 \vee X_2)$

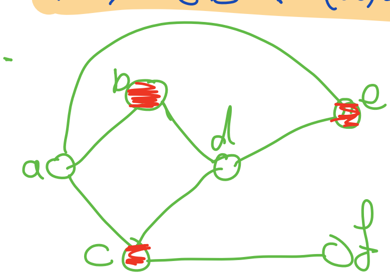
SAT $\leq_p$ 3SAT

Maximum Indepent Set (MIS)

Given an Unweighted Undirected Graph $G(V,E)$, Find The Max-Size Set of nodes S, Such that

$$\forall u, v \in S : (u,v) \notin E$$

eg.

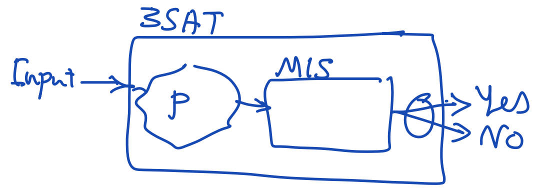


MIS = 3
{b, c, e}

Prove MIS ∈ NP-Complete.

① MIS ∈ NP

② 3SAT $\leq_p$ MIS



① MIS ∈ NP

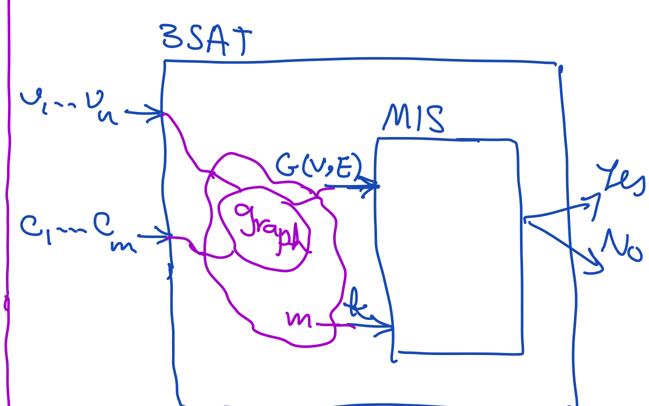
Given $G(V,E)$, k, a Set of nodes Certificate

Check if

$\forall (u,v) \in$ Certificate
  $(u,v) \in E$
    output No
output Yes

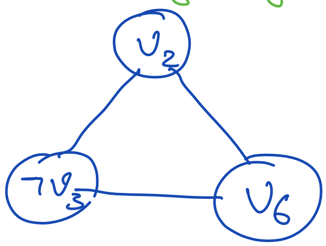
← $O(n^2)$

② 3SAT $\leq_p$ MIS



for each Clause C_i Create a Triangular Subgraph w/ 3 nodes

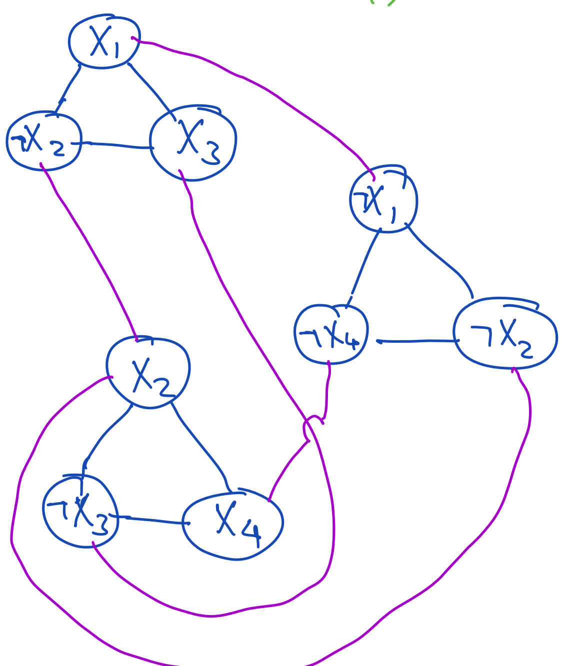
$C_i = v_2 \vee \neg v_3 \vee v_6$



$(X_1 \vee X_3 \vee \neg X_2) \wedge$

$(\neg X_1 \vee \neg X_2 \vee \neg X_4) \wedge$

$(X_2 \vee \neg X_3 \vee X_4)$



MIS $\leq_p$ 3SAT YES