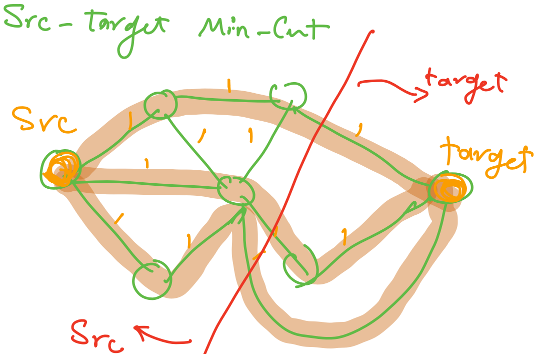
A Monte-Carlo Rand. Alg. for Min-Cut

Reminder:

Src-target Min-Cut



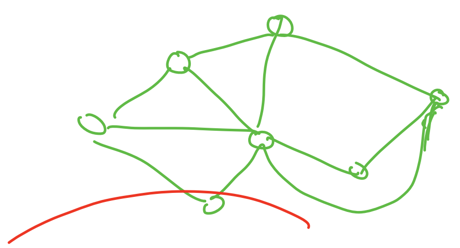
$\Longrightarrow$ Max flow = 3

Solution: find Max flow

$O(n^3)$

Min-Cut Problem:

is any cut with min # edges (no matter what the Src/target are.

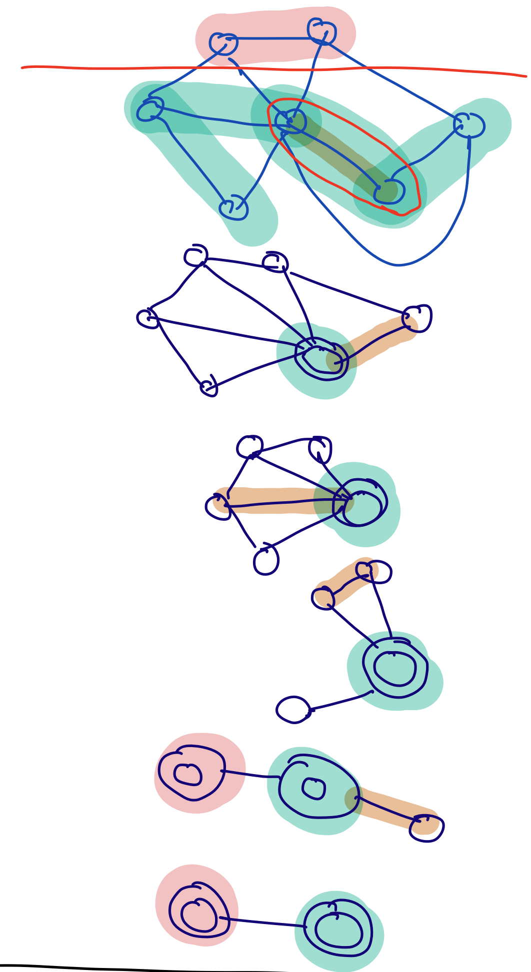


$\Rightarrow$ min Cut = 2

Solution:

- Min $\leftarrow \infty$
- for every Pair of nodes (x, y):
  - Cut $\leftarrow$ Mincut (x, y)
  - if $|Cut| < Min$
    Min $\leftarrow |Cut|$

$O(n^5)$



Rand-Min-Cut

- Select a random edge
- replace the two nodes into a super node (representing both)
- Continue until two nodes are left

Observation:

let k be the |Min-Cut|

$\Rightarrow \forall u \in V$ degree$(u) \geq k$

$\Rightarrow |E| \geq \frac{kn}{2}$

$\Rightarrow$ P(Contracting an edge from the min-Cut at first iteration)

$$= \frac{k}{|E|} \leq \frac{k}{kn/2} = 2/n$$

$P(!\text{failing at iter-1}) \geq 1 - 2/n$

at iteration 2:

$$|E| \geq \frac{k(n-1)}{2}$$

$$P(!\text{failing at iter 2}) \geq 1 - \frac{2}{(n-1)}$$

$\Rightarrow$ at iter i:

$$P(!\text{failing at iter } i) \geq 1 - \frac{2}{n-i+1}$$

$$P(\text{Success}) = \prod_{i=1}^{n-2} P(!\text{failing at iter } i)$$

$$\geq \prod_{i=1}^{n-2} \left(1 - \frac{2}{n-i+1}\right)$$

$$= \prod_{i=1}^{n-2} \left(\frac{n-i-1}{n-i+1}\right)$$

$$= \left(\frac{\cancel{n-2}}{n} \times \frac{\cancel{n-3}}{n-1} \times \frac{n-4}{\cancel{n-2}} \times \frac{n-5}{\cancel{n-3}} \times \cdots \frac{2}{\cancel{4}} \times \frac{1}{\cancel{3}}\right)$$

$$= \frac{2}{n(n-1)} > 2/n^2$$

$P(\text{Success at } i\text{th trial}) > 2/n^2$

if repeat $n^2/2$ times

$P(\text{failure at } i\text{-th trial}) < 1 - 2/n^2$

$$P(\text{failure}) < \left(1 - 2/n^2\right)^{n^2/2}$$

$$\left(1 + t/z\right)^z < e^t$$

$\Rightarrow z = n^2/2 \;,\; t = -1$

$\Rightarrow P(\text{failure}) < 1/e$

after repeating the alg. $O(n^2)$ times, it will find the min cut with a high $(> (1 - 1/e))$ Probability.