MAX-3-SAT

find an assignment to variables $x_1 \cdots x_n$, that Satisfy the max # of clauses.

Find an Alg. with $8/7$-approx. ratio for Max-3-SAT

- Randomly assign (True, false) to each variable x_i

Proof: Let y_i be 1 if clause C_i is satisfied

$$E[\text{clauses satisfied}] = E[\sum y_i] = \sum E[y_i]$$

$$X_i = \begin{cases} T & \text{Prob. } 0.5 \\ F & 0.5 \end{cases}$$

$C_i = (\square^{x_i} \vee \square^{\bar{x}_j} \vee \square^{x_k}) \rightarrow P(C_i \text{ is NOT satisfied}) = 1/8$

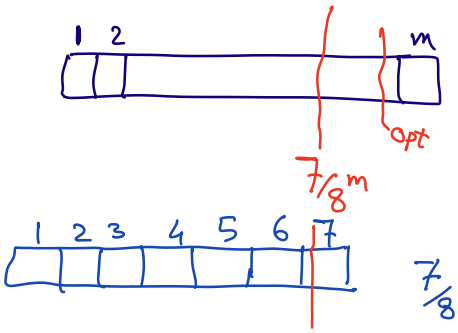
$\Rightarrow P(y_i) = 1 - 1/8 = 7/8$

$$E[\text{clauses satisfied}] = \sum_{i=1}^{m} E[y_i] = \sum_{i=1}^{m} 7/8 = 7/8\, m$$

$OPT \leq m$

$$\Rightarrow \frac{OPT}{A} \leq \frac{m}{7/8\, m} = 8/7$$

$\Rightarrow$ Given an instance of 3-SAT with 7 clauses, there always exists an assignment to $x_1 \cdots x_n$ S.t. all clauses are satisfied



$\frac{7}{8} \times 7 \simeq 6.1$

There should always exist a number Larger (or equal) to the Average

⇒ There should exist an assignment that satisfies 7 clauses otherwise the average could have not been 6.1