Integer Programming

given a set of variables $x_1 \cdots x_n$, each being integer

given a set of linear constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\ddots$$
$$Ax \leq b$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

given a linear function $f(x)$,

find an assignment to the varibles that max/min $f(x)$ s.t. $Ax \leq b$

Linear Programming

given a set of variables $x_1 \cdots x_n$, each being a real number

$$Ax \leq b$$

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e.g. $x_1, x_2 \in \mathbb{R}$

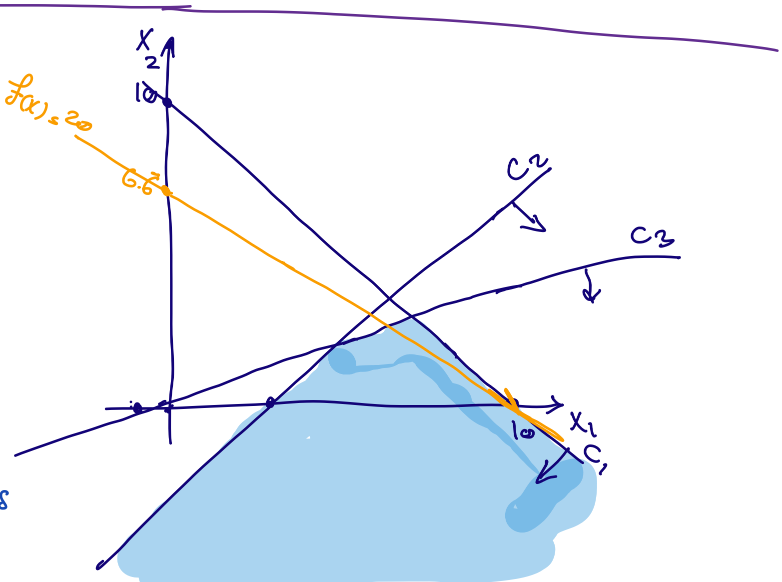
max $2x_1 + 3x_2$

s.t.

c_1: $x_1 + x_2 \leq 10$

c_2: $x_1 - x_2 \geq 3$

c_3: $2x_2 - x_1 \geq 1$



observations:

1. The optimal solution is a corner point
2. The search space is always convex

Integer Programming is NP-Complete

$VC \leq_p IP$

Linear Programming is NP-Complete?

2D (x_1, x_2):

- $O(m^2)$ Corner Points

$O(m^3)$

- for each intersection

$O(m)$ if it belongs to the valid Search Space

Compute $f(x)$

return the intersection with max/min $f(x)$

for 3D (x_1, x_2, x_3):

every Corner point is the intersection of 3 planes

$\rightarrow O(m^3)$ Corner Points

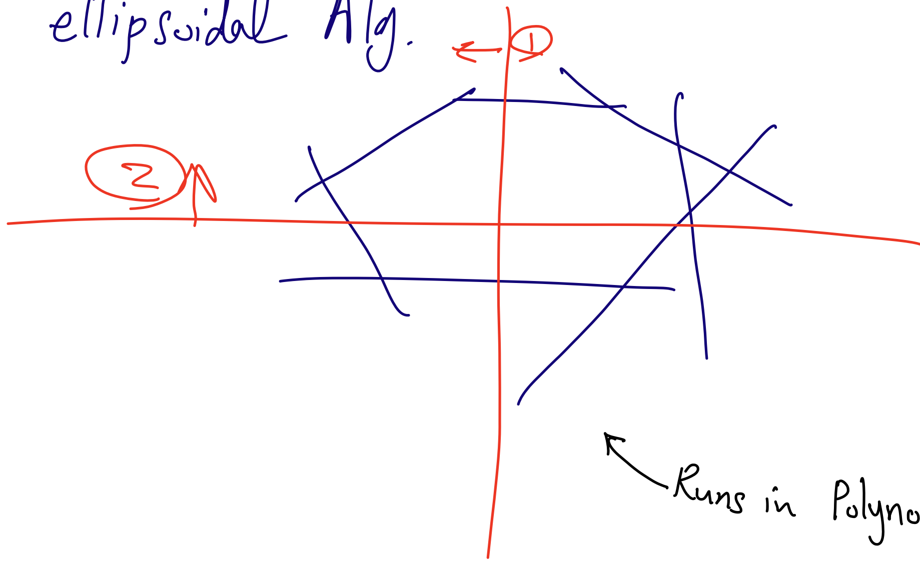
$\rightarrow O(m^4)$

for nD $(x_1 \dots x_n)$

n hyper Planes $\rightarrow O(m^n)$ Corner Points

$\rightarrow O(m^{n+1})$

ellipsoidal Alg.



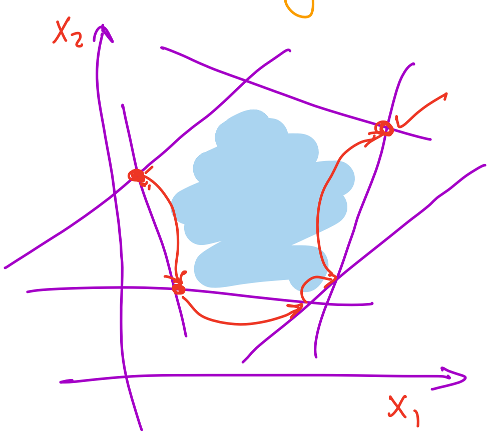
Runs in Polynomial Time

$\hookrightarrow$ $LP \in P$

Practical Algorithm:

Simplex (in Practice is polynomial, worst-Case exponential)

$\hookrightarrow$ Hill Climbing



$\max(x_1)$

Step 1: find a Corner Point

Step 2: Until the opt. is found move to a neighbor with a better obj. value.