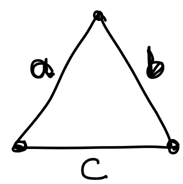
Metric-TSP { 2-approx. alg.
1.5- " . alg.
4/3- approx. alg.

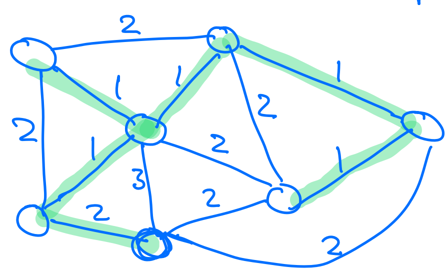
Metric-Space: edge weights follow the Triangular inequality



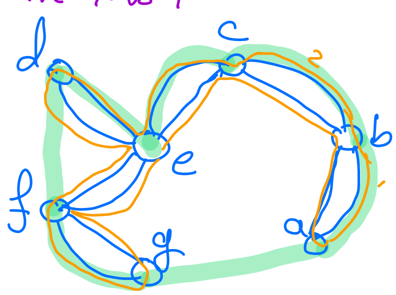
$c \leq a+b$

2-approx. alg. for M-TSP

Step 1: Find the min Spanning Tree on G (Complete Graph)



Step 2: double every edge in MST



The resulting Graph is Eulerian

Step3:
follow the Eulerian Circuit while Taking Short Cuts

Claim: the alg. has an approx-ratio of 2.

- $|MST| \leq OPT$
- $|E\text{-}Graph| = 2|MST|$
- $Approx = |Cycle| \overset{*}{\leq} |E\text{-}Graph|$

* Triangular-Inequality

$Approx \leq 2|MST|$

$\leq 2\,Opt$

$\Rightarrow \dfrac{Approx}{opt} \leq 2$

1.5-approximation Algorithm.

Step 1: Find the MST

Step 2: Convert the Tree into a Eulerian Graph, using the min matching routine

Step 3: follow the Eulerian Circuit while Taking Short Cuts.

Observation:

The # of odd-degree nodes is even.

$\sum_{v_i} deg(v_i)$ is even

bcz every edge is Counted Twice

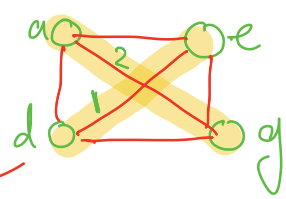
$$\sum_{all} deg(v_i) = \sum_{odd} deg(v_i) + \sum_{even} deg(v_i)$$

(Even) $\qquad$ (Even)

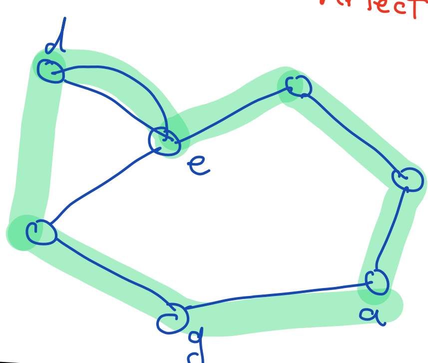
$\Rightarrow \sum_{odd} deg(v_i)$ is even

$\Rightarrow$ The # of odd-degree nodes is even

e.g.



find the min-Cost ~~Perfect~~ Matching



- The edges added by the min-Cost matching is at most: $Cost(M) \leq OPT/2$

- Proof.

• Let OPT be the opt. Cycle by TSP.

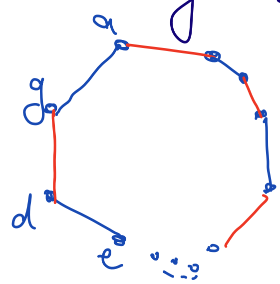
• Let V' be the Set of odd-degree nodes on the MST.

→ Let τ be the tour on V' following opt. Cycle, while taking Short-cuts.

→ $Cost(\tau) \leq OPT$

τ is the Union of two Perfect matchings on V'

Let Blue and Red be the two matchings as shown below:



- Suppose Blue is the min Cost matching

$OPT \geq Cost(\tau)$

$Cost(\tau) = |Blue| + |Red|$

$|Blue| \leq |Red|$

$\Rightarrow OPT \geq 2\,|Blue|$ ✓

- Approx-ratio of the Alg. is 1.5

Proof:

$|E\text{-}Graph| = |MST| + |Blue|$

$|MST| \leq OPT$

$|Blue| \leq OPT/2$

$|E\text{-}Graph| \leq 3/2\, Opt$

$Approx \leq |E\text{-}G| \leq 3/2\, Opt$

$$\frac{Approx}{OPT} \leq 3/2 = 1.5$$