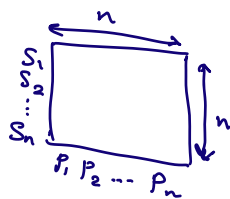


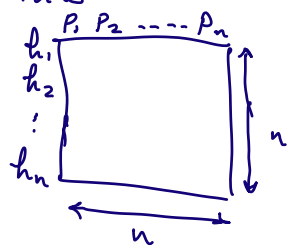
Stable Matching Problem

Input: $\begin{cases} n \text{ hospitals } H \\ n \text{ Students } S \end{cases}$

Pref. of Students



Pref. of Hospitals



Goal (Output): ?

Find a Stable (Perfect) Matching

→ No. unstable pair
→ 1-1 matching

$h-s$ is unstable iff

- h prefers s over their current match
- $s \sim h \sim \sim \sim \sim$

Gale-Shapley Alg.

while ($\exists h$ that is unmatched)

- h proposes to their most preferred student s^* that has not rejected their offer
 - if (s is unmatched)
 - add $s-h$ to matches
 - else if (h is preferred over the current match of s)
 - remove $h'-s$
 - add $h-s$ to matches
 - else reject the proposal
- return matches

- Q1: Does the alg. terminate? (Correctness)

observation1: hospitals propose in order until they exhaust the List.

observation2: It is 1-1 mapping. there should exist at least 1 student for each hospital.

⇒ The Alg. terminates

- Q2: Run-Time? (Max. # of proposals made)?

n^2 : Every hospital proposes to at most n students & there are n hospitals

- Q3: Does it produce a Perfect Matching?

- Proof by Contradiction.

- Q4: Does it produce a Stable matching.

Consider an $h-s$ ~~unstable pair~~ ^{unstable pair}

- h did not propose to s
- h prefers their current match over s .
- not unstable

- h proposed, s rejected
- s has a proposal they prefer more over h
- not unstable

✗

- Hospital Optimizing

- hospitals are always matched to their best possible student

Proof:

$$s-h \in M$$

but h prefers s' over s

if h prefers $s' \Rightarrow h$ proposes to s' first

$\rightarrow s'$ rejects h if they Recv. offer from h' they prefer over h

$$h: s' > s$$

$$s': h' > h$$

$$\Rightarrow h - s'$$

is an unstable pair

$\Rightarrow M$ is not a stable matching ❌

- Student pessimaling

- Students receives their most valid match.

Proof (by contradiction)

$$s-h \in M^*$$

- assume there exist a hospital h' , where $s-h'$ would be valid and s prefers h over h'

$$s: h > h'$$

- Let M be a stable matching where $s-h' \in M$

- h prefers s over s'
 $\hookrightarrow$ hospital optimality

$\Rightarrow h-s$ is unstable pair in M

$\Rightarrow M$ is not stable ❌