$f(n) = 5n^5 + 6n^4 + 8$ operations

Big-O notation

a function $f(n) = O(g(n))$

if $\exists$ Constants c, n_0, Such that

$\forall n > n_0$

$$0 \leq f(n) \leq C\,g(n)$$

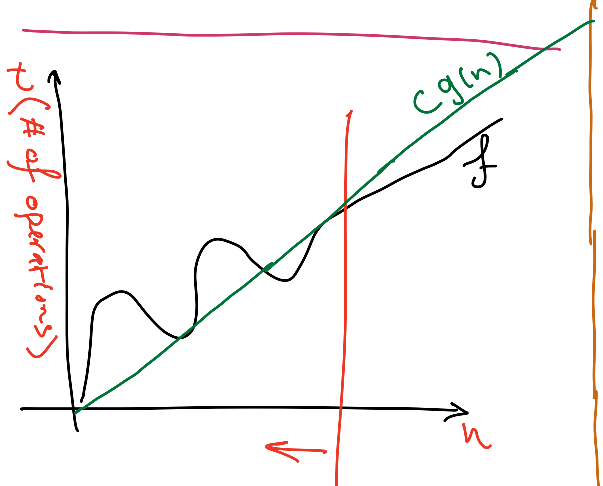
e.g. : $g(n) = n^5$

is $f(n)$ in $O(g(n))$?

$n_0 \geq 10$

$C \geq 12$ ✓

$\Rightarrow 5n^5 + 6n^4 + 8 = O(n^5)$



$f(n) = O(g(n))$

Big-Omega

a function $f(n) = \Omega(g(n))$

if $\exists$ Constants c, n_0, Such that

$\forall n > n_0$

$$f(n) \geq C\,g(n) \geq 0$$

e.g. : $g(n) = n^5$

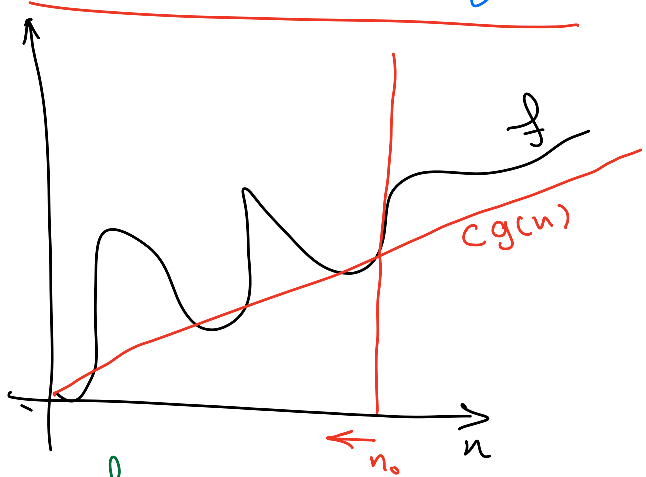
is $f(n)$ in $\Omega(g(n))$?

$n_0 \geq 1$

$C = 1$

$5n^5 + 6n^4 + 8 \geq Cn^5$

✓



$f(n) = \Omega(g(n))$

Big – Theta

a function $f(n)$ is in $\Theta(g(n))$

if $\exists$ Constants c_1, c_2, n_0,

s.t.

$\forall n > n_0$

$$c_1 g(n) \leq f(n) \leq c_2(g(n))$$

$\forall n \geq 10$

$$1 n^5 \leq 5n^5 + 6n^4 + 8 \leq 12 n^5$$

$$\Rightarrow f(n) = \Theta(g(n))$$

Observation:

Any function in form

$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_0$$

$$\Rightarrow \boxed{f(n) = \Theta(n^k)}$$

Observation:

if $f(n) = \Theta(g(n))$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = C$$

Constant

$$\lim_{n \to \infty} \frac{5n^5 + 6n^4 + 8}{n^5} = 5$$

Questions:

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

is $f(n)$ in $O(g(n))$?

$\Rightarrow$ Yes

e.g. $\lim \frac{5n^5}{n^6} = 0$

$$5n^5 = O(n^6)$$

$\Rightarrow$ If $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$,

$$f(n) = o(g(n))$$

Little-oh

If $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$

, is $f(n) = \Omega(g(n))$?

$\Rightarrow$ Yes

$$5n^5 = \Omega(n^4)$$

if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$,

$$f(n) = \omega(g(n))$$

Little-omega

Constant - Time Alg.

$f(n) = O(1)$

- Random Access to Array

- find the first 1000 prime numbers.

  $\hookrightarrow O(1)$