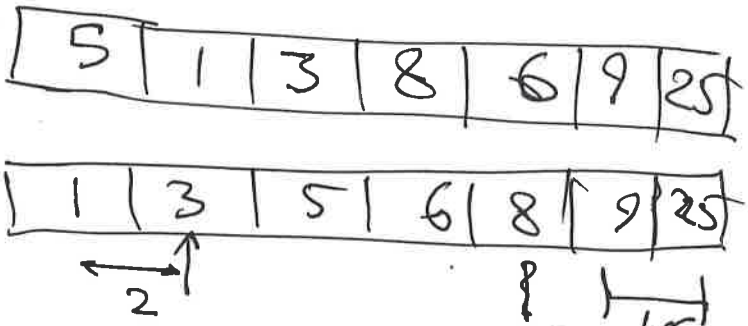
① Time Stamps

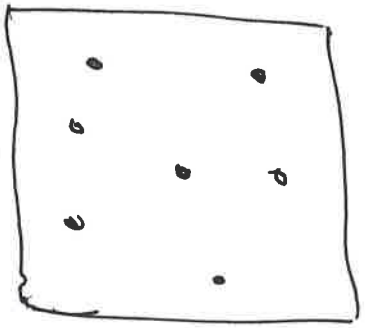
← Given a set of Time Stamps of events, find the longest silent interval



① Sort ← $O(n \log n)$ } $O(n \log n)$

③ Pass ← $O(n)$

Closest Pair



$min = \infty$

for each Pair P_i, P_j
  if $d(P_i, P_j) < min$
    $min = d(P_i, P_j)$

$O(n^2)$

↑

Given a set of Points on a plane (map) find the closest Pair

Points $P_1, P_2, P_3, \ldots, P_n$

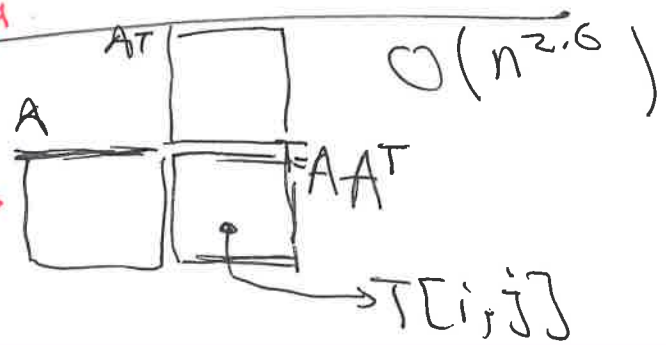
Sets $S_1, S_2, \ldots, S_n$

eg. $S_i = \{P_{10}, P_8\}$ elements

	P_1	P_2	...	P_n
S_1	1	0	...	0
S_2	0	1	...	1
⋮	⋮	⋮	⋮	⋮
S_n	1	1	...	1

$O(n^3)$

Given a Set of Points $P_1 \ldots P_n$ and a Collection of Sets $S_1 \ldots S_n$, find all disjoint Pairs of Sets



$O(n^{2.6})$

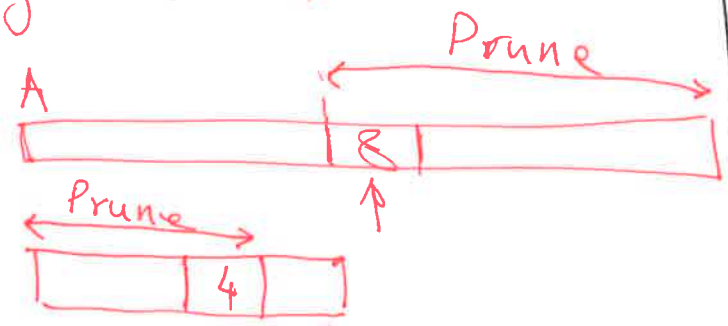
② Binary Search.

- Sorted list A
- key x
- if A Contains x

A

1	3	4	6	8	9	10	12

Key x = 5



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A does not Contain 5

$$T(n) = \begin{cases} 1 + T(n/2) & \\ 1 & n=1 \end{cases}$$

$$T(n) = 1 + 1 + T(n/2^2)$$

$$= 1+1+1+T(n/2^3)$$

$$= \underbrace{1+1+\cdots+1}_{k} + T(n/2^k) = \Theta(\log n)$$

$2^k = n \Rightarrow k = \log n$

Merge-Sort

$$T(n) = 2T(n/2) + n$$

$$= n + 2\left[n/2 + 2T(n/2^2)\right]$$

$$= 2n + 2^2 T(n/2^2)$$

$$= kn + 2^k T\left(\frac{n}{2^k}\right)$$

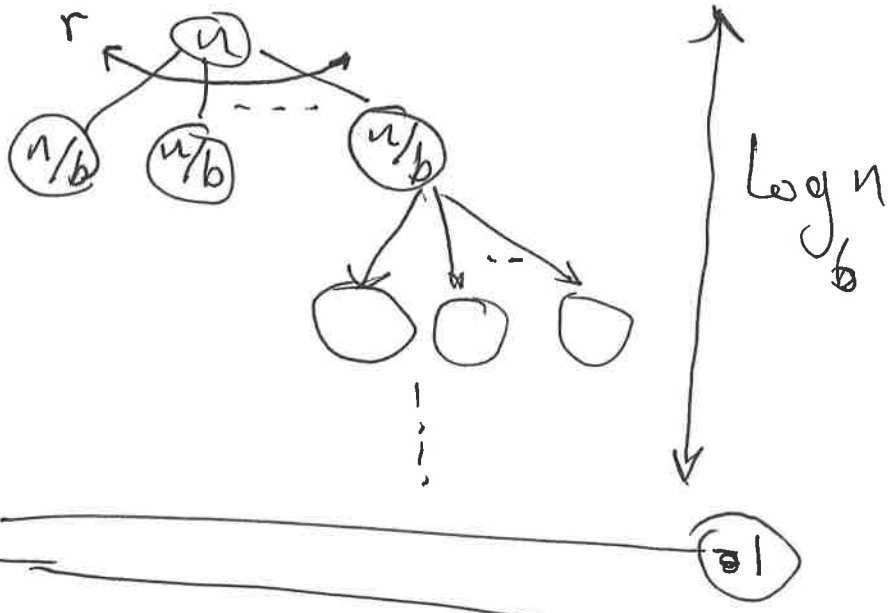
$k = \log(n)$

$$= n\log n + n$$

$$= \Theta(n \log n)$$

```
M-Sort(A, l, h):
    if (l == h)
        return
    mid = ⌊(l+h)/2⌋
    M-Sort(A, l, mid-1)
    M-Sort(A, mid, h)
    Merge(A, l, mid, h)
```

(3)



$$T(n) = aT(n/b) + \Theta(n^d)$$

$a < b^d$: $T(n) = \Theta(n^d)$

$a = b^d$: $T(n) = \Theta(n^d \log n)$

$a > b^d$: $T(n) = \Theta(n^{\log_b a})$

$$T(n) = T(n/2) + 1$$

$a = 1, b = 2, d = 0$

$a = 1, b^d = 1$

$T(n) = \Theta(n^0 \log n) = \Theta(\log n)$

$$T(n) = 2T(n/2) + n$$

$$= \Theta(n \log n)$$