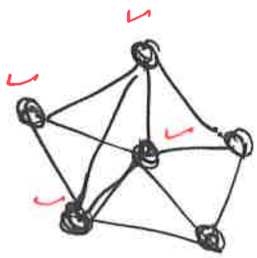
Clique:

Given a graph $G(V,E)$ find the Max Number of vertices that Construct a Complete graph (Sub graph)

i.e.

$\forall\, u \in S,\ v \in S \Rightarrow (u,v) \in E$

e.g.



Clique ∈ NP-Complete ?

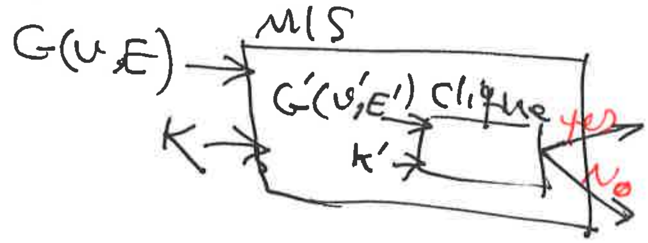
(A) Clique ∈ NP.

Certificate: given a subset of nodes, is it a clique of size k

→ between every pair of nodes in the set check if there is an edge → $O(n^2)$

(B) Reduction

MIS $\le_p$ Clique



$K' = K$

$V' = V$

$\forall\, (u,v) \notin E$, add $(u,v) \in E'$

* A clique of size k in graph G' shows an Indep. Set in G

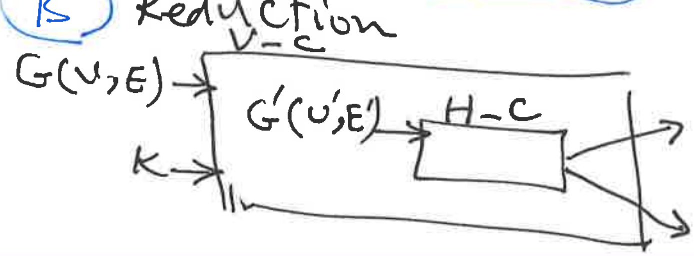
Hamiltonian Cycle (H-C)

Given a graph $G(V,E)$ is there a simple cycle that passes through all nodes.

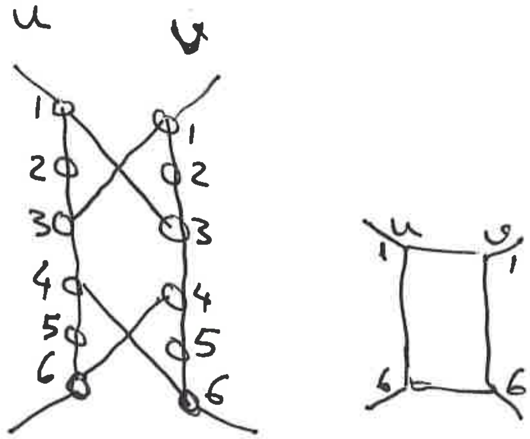
(A) H-C ∈ NP

Certificate: given a sequence of nodes check if it is a Hamiltonian-Cycle. → $O(n)$

(B) Reduction

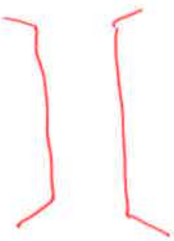


$\forall$ edge $(u, v) \in E$



if only $\underline{\underline{u}}$ is Selected

if only $\underline{\underline{v}}$ is Selected



both $\underline{u}$ and $\underline{\underline{v}}$ are Selected

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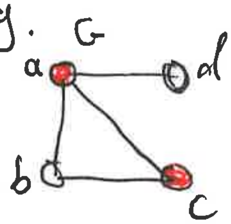
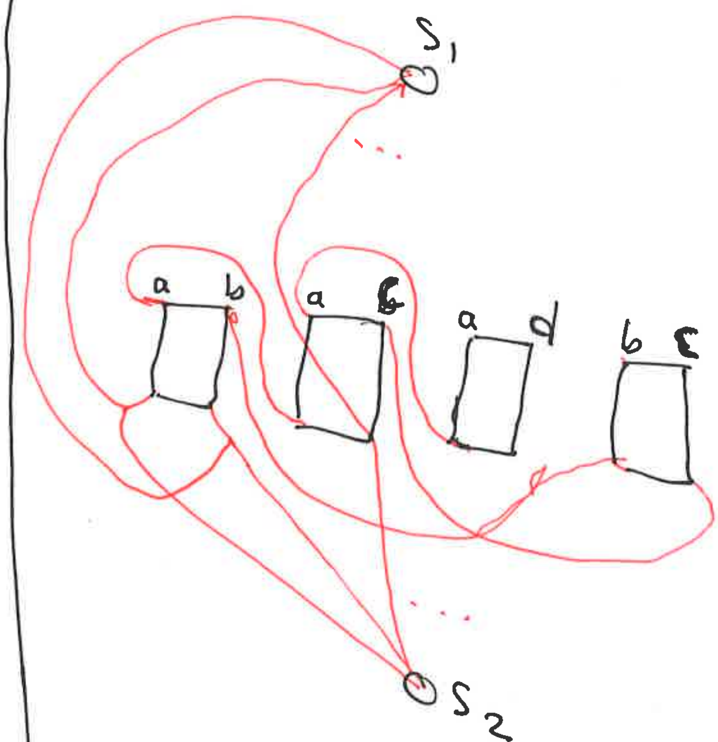
eg. G



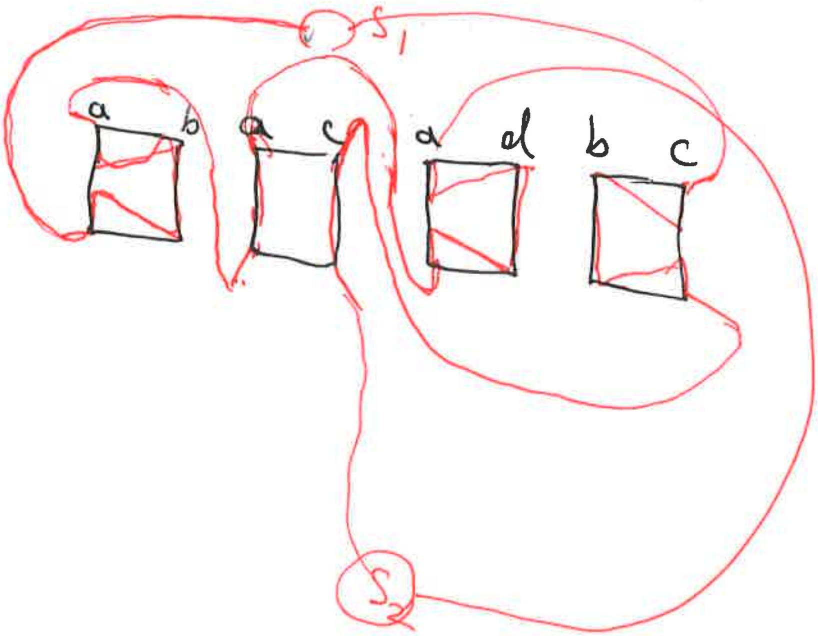
$k = 2$



add k dummy nodes $S_1 \cdots S_k$

all the free $\underline{\underline{\text{wires}}}$ get Connected to all $S_1, S_2, \dots, S_k$

The answer to the decision_vc is yes iff G' has a h-c.



Traveling Salesman Person (TSP)

Given a <u>weighted</u> <u>complete</u> graph find the simple cycle with min cost.

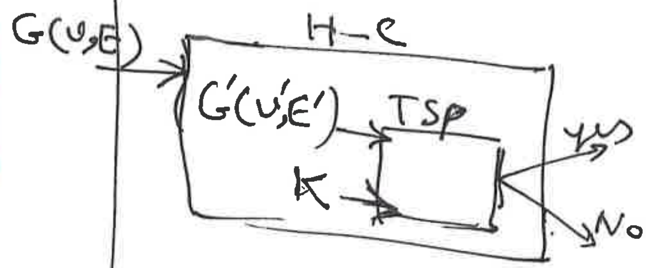
TSP $\in$ NP-Complete ?

Ⓐ TSP $\in$ NP

Certificate: a seq. of nodes and a value K, is this cycle of cost at most K ? $\rightarrow O(n)$

Ⓑ Reduction

H-C $\leq_p$ TSP



$V' = V$

$\forall (u,v) \in E,\ W^{G'}_{u,v} = 1$

$\forall (u,v) \notin E,\ W^{G'}_{u,v} = \infty$

$K = n$