$VC \equiv_p MIS$

if S is a MIS

$\forall (u,v) \in E$, $u \notin S$ or $v \notin S$

$\Rightarrow u \in V-S$ or $v \in V-S$

$\Rightarrow V-S$ is a vertex Cover

$\Rightarrow VC \leq_p MIS$ (A)

---

if $V-S$ is a vertex Cover

$\forall$ ~~xxx~~ $u \in S, v \in S$

if $(u,v) \notin E$

$\Rightarrow S$ is an Indep. Set

$\Rightarrow MIS \leq_p VC$ (B)

(A), (B) $\Rightarrow VC \equiv_p MIS$

– Reduction Strategy #2:

Reducing a General problem to a Specific problem.

Set Cover

Given

a universe of items

$U = \{I_1, \ldots, I_n\}$

and a collection of sets

$S = \{S_1, S_2, \ldots, S_m\}$

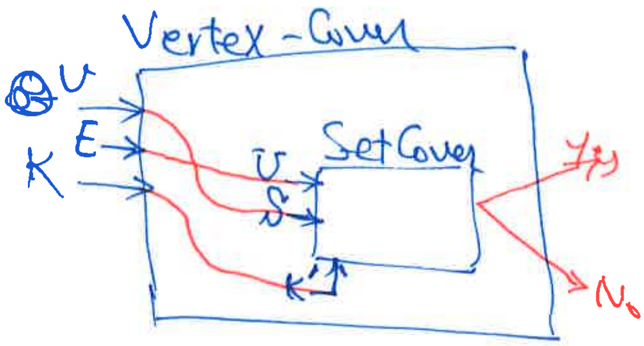
$S_i \subseteq U$, $\bigcup_{\forall i \in [1,m]} S_i = U$

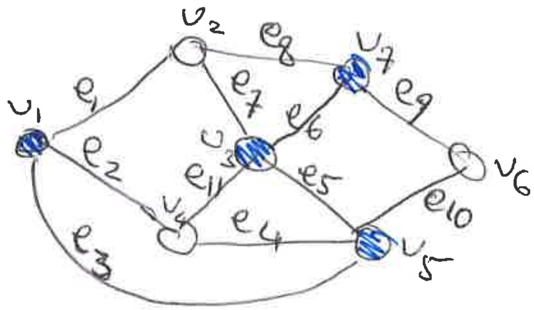
find $\underbrace{\text{Subset of } S}_{S'}$

s.t. $S' \subseteq S$ and $\bigcup_{\forall s \in S'} s = U$

$\min |S'|$

$SC \in NP$ ✓





$U = \{e_1, e_2, \dots, e_{11}\}$

$v_1 = \{e_1, e_2, e_3\}$

$v_2 = \{e_1, e_7, e_8\}$

$v_3 = \{e_7, e_6, e_5, e_{11}\}$

...

$v_7 = \{e_6, e_8, e_9\}$

$S = \{v_1, v_2, \dots, v_7\}$

e.g. $k = 4$

$\{v_1, v_3, v_5, v_7\}$

Set Cover $\longrightarrow$ Yes

Vertex_Cover is reduced to Set_Cover

$VC \leq_p SC$

Reduction Strategy #3: designing a gadget

3-SAT Problem:

$C_1 \wedge C_2 \wedge \dots \wedge C_n$

$(\bar{v}_1 \vee v_2 \vee v_5) = C_1$