Complexity Classes of Problems

P: there exists a polynomial Solution for the problem

NP: B

NP-Complete: B, C

NP-hard: C

For every Problem:

A) Optimization Version:

$\exists$ objective function f

min/max f

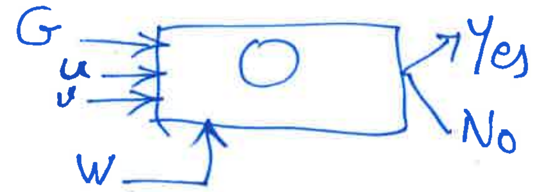
B) verification version:

Given a $\boxed{\text{certificate}}$ (a solution) check if it is valid

C) Decision Version:

given a value $\boxed{\underline{V}}$, does the Problem have a solution where $\boxed{f = V}$

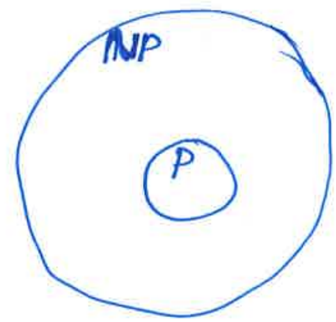
e.g.: given $G(V,E)$, is there a Path of weight w between (u,v)



Yes: between $u \rightsquigarrow v$ there is a path of weight w

---

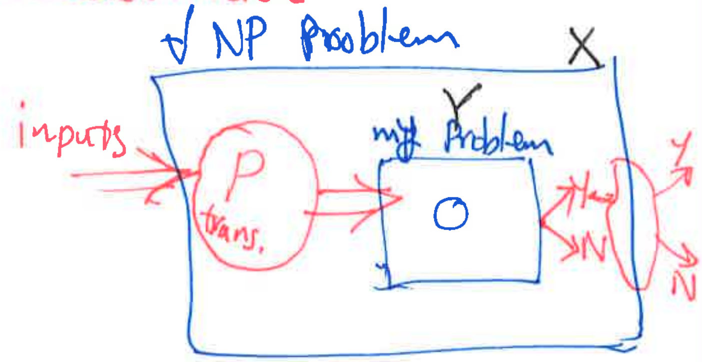
NP: a probem that can be verified in P.



$P \subseteq NP$ ✓

$P \stackrel{?}{=} NP$ Unknown!!

Decision Version



Reduction

$Y \leq_p X$

Strategies for reduction

① Show the Problems are Equivalent

Eg. VC : Vertex_Cover

MIS : Max. Indep. Set

VC :

(opt version)

Given a graph $G(V,E)$

find $\underline{\underline{\text{min}}}$ $S \subseteq V$

s.t.

$\forall$ Edge $(u,v) \in E$

$u \in S$ or $v \in S$

(verification version)

Given $G(V,E)$, $S \subseteq V$

verify if S is a vertex_Cover of size $|S|$

Th: $VC \subseteq NP$?

Yes, because verification Can be done in $\underline{\underline{P}}$

(V-C) (Decision Version)

Given $G(V,E)$ and a value $\underline{\underline{K}}$,

does G has a vertex Cover of size at most K

MIS : Max. Indep Set

(opt. version)

Given $G(V,E)$,

find max $S \subseteq V$

s.t.

$\forall u,v \in S$

$(u,v) \notin E$

$VC \overset{?}{\equiv}_p MIS$

a VC of size K is an indep. set of size $|V| - K$