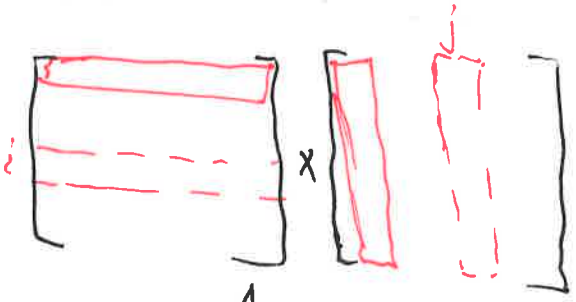
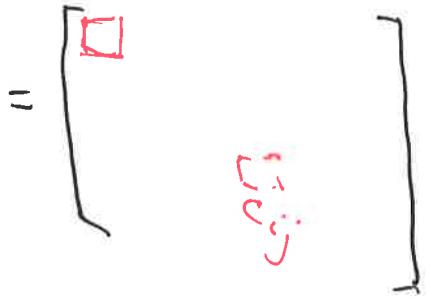
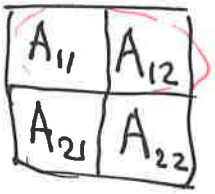


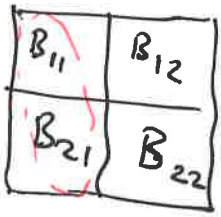


$O(n^3)$: Brute force

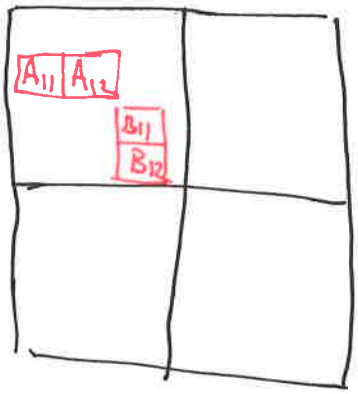
A



B



C =



8 multiplications, $n/2 \times n/2$

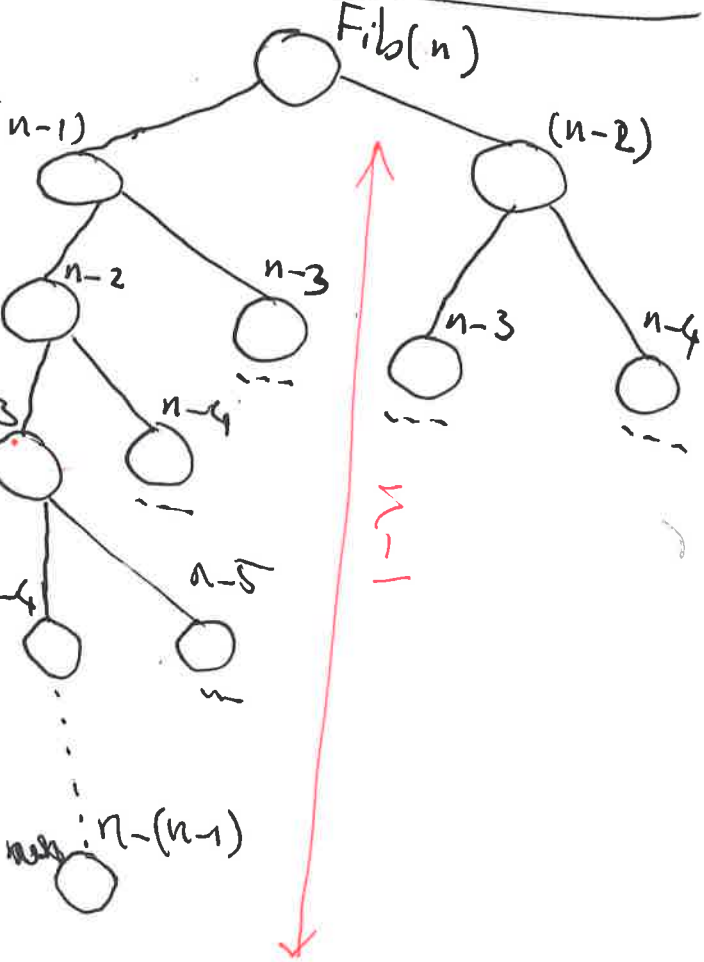
$T(n) = 8T(n/2) + \Theta(n^2)$

$\Theta(n^{\log_2 8}) = \Theta(n^3)$

<0, 1, 1, 2, 3, 5, 8, 13, ...>

$$F_n = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ F_{n-1} + F_{n-2} & n \geq 2 \end{cases}$$

```
Fib(n):
  if (n < 2)
      return n
  Fib(n-1) + Fib(n-2)
```



(4)

F_i : The Seri from F_0 to F_i

$\langle 0, 1, 1, 2, 3, \ldots, \text{ith} \rangle$

$E(F^{(i)}) = \langle 1, 2, 3, \ldots, i+1\text{th} \rangle$

$$F_n = F_{n-1} + F_{n-2}$$

$$(E^2 - E - 1) F_n = 0$$

$$(E - \phi)(E - \hat{\phi}) = 0$$

$E^2 - E - 1 = 0$, $\Delta = b^2 - 4ac$

$= 1 + 4 = 5$

$\phi_{1,2} = \frac{-2 \pm \sqrt{5}}{2}$

$$F_n = C_1 \phi^n + C_2 \hat{\phi}^n$$

$$\begin{cases} F_0 = C_1 \phi^0 + C_2 \hat{\phi}^0 = C_1 + C_2 = 0 \\ F_1 = C_1 \phi + C_2 \hat{\phi} = 1 \end{cases}$$

$C_1 = 1/\sqrt{5}$, $C_2 = -1/\sqrt{5}$

```
Fib(n)
    if (n < 2) return n
    a = 0, b = 1
    for i = 2 to n
        c = a + b
        a = b
        b = c
    return c
```